Unit 3 Detailed study 2: Investigating structures and materials

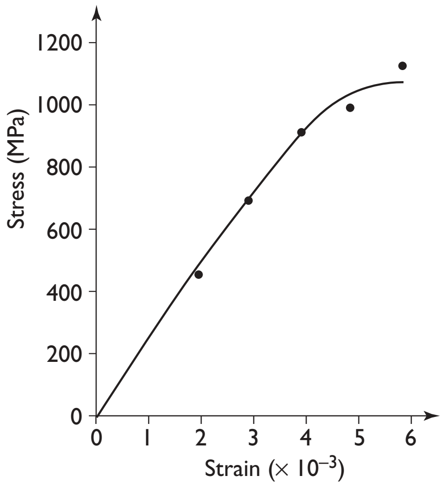
Chapter 8

Investigating materials

1. (a) Reading from the graph,
   $\Delta l = 0.60$ mm
   (b) Young's modulus is determined from the slope of the σ vs ε graph. However, in this case use the relationship $\sigma = Y \times \varepsilon$:
   $$Y = \frac{\sigma}{\varepsilon} = \frac{F \times L}{A \times \Delta l}$$
   $$= \frac{150 \times 0.100}{50 \times 10^{-6} \times 0.6 \times 10^{-3}}$$
   $$= 500 \times 10^{6}\ \text{Pa}$$
   $$= 500\ \text{MPa}$$
2. Estimate the area of your feet. Taking a mass of 60 kg supported on two feet, each approximately 25 cm × 6 cm, the stress would be
   $$\frac{60 \times 10}{2 \times 0.25 \times 0.06} = 2 \times 10^{4}\ \text{Pa.}$$
   The stress in the column is $\frac{4000}{\pi(0.100)^2} = 1.3 \times 10^{5}$.
   The column causes the larger stress.
3. (a) Strain = ε
   $$= \frac{\Delta l}{L}$$
   $$= \frac{100}{10\,000}$$
   $$= 0.01$$
   (b) Stress $= \sigma = \frac{F}{A}$
   $$= \frac{50 \times 10^{3}}{\pi \times (0.004)^2}$$
   $$= 9.9 \times 10^{8}\ \text{Pa}$$
4. $Y = \frac{\sigma}{\varepsilon}$
   $$= \frac{80\ \text{MPa}}{0.002}$$
   $$= 4 \times 10^{4}\ \text{MPa}$$
   $$= 40\ \text{GPa}$$
5. (a) Young's modulus is given by the slope of the σ vs ε graph. A has the larger Young's modulus as it is the steepest.
   (b) B
   (c) Toughness is given by the area under the σ vs ε graph. B is therefore the tougher material.
   (d) B exhibits more plastic behaviour and is therefore more ductile.
   (e) A
6. (a) Stress $= \sigma = \frac{F}{A}$
   $$= \frac{100 \times 10^{3}}{\pi \times 0.100^2}$$
   $$= 3.2 \times 10^{6}\ \text{Pa}$$
   $$= 3.2\ \text{MPa}$$
   (b) $\varepsilon = \frac{\Delta l}{L} = \frac{0.05 \times 10^{-3}}{5} = 1 \times 10^{-5}$
7. $\sigma = Y \times \varepsilon$
   $$= 110 \times 10^{9} \times 3 \times 10^{-4}$$
   $$= 3.3 \times 10^{7}\ \text{Pa}$$
   $$= 33\ \text{MPa}$$
8. From:
   $$\sigma = \frac{F}{A}$$
   $$= Y \times \varepsilon$$
   $$= \frac{Y \times \Delta l}{L}$$
   $$\Rightarrow \Delta l = \frac{F \times L}{A \times Y}$$
   $$= \frac{15 \times 10^{3} \times 5}{\pi \times (0.001)^2 \times 200 \times 10^{9}}$$
   $$= 0.119\ \text{m}$$
   $$\cong 120\ \text{mm}$$
9. (a) Using stress $= \sigma = \frac{F}{A}$,
   where $A = 12.7 \times 12.7 \times 10^{-6}\ \text{m}^2 \cong 161 \times 10^{-6}\ \text{m}^2$,
   and strain $= \varepsilon = \frac{\Delta l}{L}$, where $L = 5.08 \times 10^{-2}$ m

Stress (MPa)	Strain
0	0
449	1.97×10^{-3}
673	2.95×10^{-3}
898	3.94×10^{-3}
999	4.92×10^{-3}
1170	5.91×10^{-3}

(b)



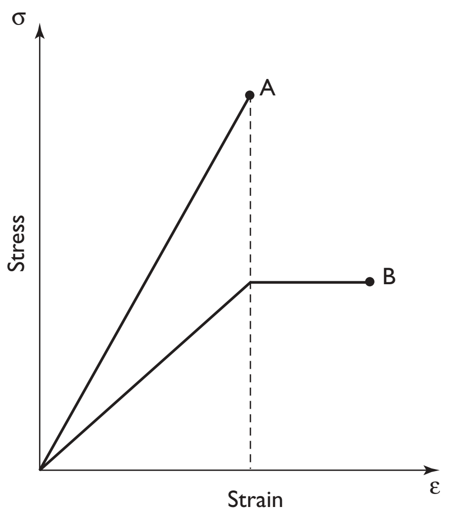
$Y = \dfrac{680 \text{ MPa}}{3 \times 10^{-3}}$

$= 227 \times 10^3$ MPa

$= 227$ GPa

(c) A strain of 0.5% = 0.005 is beyond the elastic limit. Therefore use the graph rather than the relationship $\sigma = Y \times \varepsilon$. From the graph $\sigma \cong 1030$ MPa.

10. (a)



Area under curve A = area under curve B.

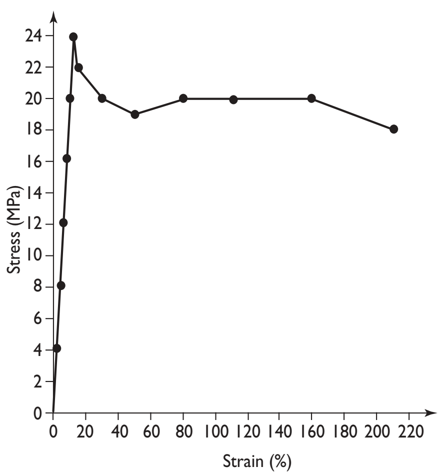
(b) As A and B are equally tough, the areas under the graphs must be equal. This shows that A is stiffer by a factor of 2.

11. $\sigma = Y \times \varepsilon$

$= 110 \times 10^3 \text{ MPa} \times 5 \times 10^{-4}$

$= 55$ MPa

12. (a)



(b) It is ductile because of its noticeable plastic behaviour.

(c) From the slope of the linear portion of the graph before yielding, $Y = \dfrac{24 \text{ MPa}}{12 \times 10^{-2}} = 200$ MPa.

(d) When stretched to twice its original length, $\Delta L = L$; that is, $\varepsilon = \dfrac{L}{L} = 1 = 100\%$.

The stored energy is found from the area under the σ vs ε graph.

Stored energy $\cong 20 \times 10^6 \times 1 = 2 \times 10^7 \text{ J m}^{-3}$

13. The stiffness of fishing line (a) is constant but the stiffness of (b) and (c) changes. When the stress–strain graph is steeper, the fishing line is stiffer. In each case the fisherman will feel the same increasing force, assuming that the stress in each case is applied at a constant rate.

14. Energy $= \frac{1}{2}\sigma \times \varepsilon$

$= \frac{1}{2} \times \dfrac{54 \times 10^6 \times 2 \times 10^{-3}}{5}$

$= 1.1 \times 10^4 \text{ J m}^{-3}$

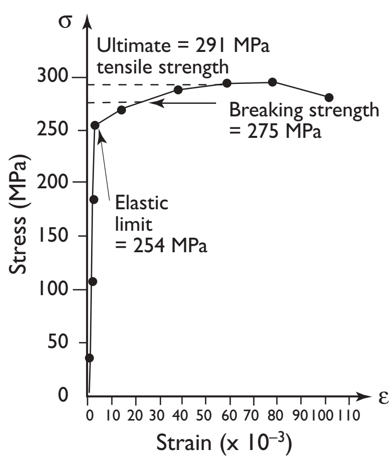
15. (a) Using stress $= \sigma = \dfrac{F}{A}$,

where $A = \pi\left(\dfrac{12.5 \times 10^{-3}}{2}\right)^2 \text{ m}^2 \cong 122 \times 10^{-6} \text{ m}^2$,

and strain $= \varepsilon = \dfrac{\Delta l}{L}$, where $L = 50.00 \times 10^{-3}$ m

Stress (MPa)	**Strain**
0.0	0.0000
36.7	0.0004
109	0.0014
182	0.0026
254	0.0036
272	0.0150
287	0.0400
291	0.0600
291	0.0800
275	0.1030

(b) and (c)



(d) (i) $\sigma_Y = \dfrac{31.2 \times 10^3}{122 \times 10^{-6}} = 254$ MPa

(ii) The ultimate tensile strength is given by the largest stress = 291 MPa.

(iii) Breaking strength = stress at fracture = 275 MPa

16. (a) The minimum force to cause plastic deformation will be just larger than the force at yield.
That is,
$F = \sigma_Y \times A$
$= 55 \times 10^6 \times \pi \times (1.5 \times 10^{-3})^2$
$\cong 3.9 \times 10^2$ N

(b) $\Delta l = \varepsilon \times L$
$= \dfrac{\sigma_Y \times L}{Y}$
$= \dfrac{55 \times 10^6 \times 40 \times 10^{-3}}{76 \times 10^9}$
$= 2.9 \times 10^{-5}$ m
$= 2.9 \times 10^{-2}$ mm

(c) $F = \sigma \times A = 125 \times 10^6 \times \pi \times 0.0015^2 = 884$ N

17. $\varepsilon = \dfrac{\sigma}{Y} = \dfrac{90 \times 10^6}{75 \times 10^9} = 1.2 \times 10^{-3}$

$\Delta l = \varepsilon \times L = 1.2 \times 10^{-3} \times 1 = 1.2 \times 10^{-3}$ m = 1.2 mm

18. (a) $\sigma = \dfrac{F}{A} = \dfrac{6 \times 10^3}{\pi \times (1.5 \times 10^{-3})^2} = 850$ MPa

(b) $\varepsilon = \dfrac{\Delta l}{L} = \dfrac{0.4}{500} = 8 \times 10^{-4}$

(c) $Y = \dfrac{\sigma}{\varepsilon} = \dfrac{850 \times 10^6}{8 \times 10^{-4}} = 1.1 \times 10^{12}$ Pa

(d) From the area under the graph of force vs extension,
strain energy $= \frac{1}{2} \times 6.0 \times 10^3 \times 0.4 \times 10^{-3}$
$= 1.2$ J

(e) Total energy $= \frac{1}{2} \times 6.0 \times 10^3 \times 0.4 \times 10^{-3}$
$= 1.2$ J

19. (a) (i) Tension
(ii) Shear
(iii) Tension
(iv) Compression
(v) Tension
(vi) Tension
(vii) Tension
(viii) Compression
(ix) Tension
(x) Compression

(b) (i) Nylon — tough, elastic, stiff
(ii) Steel — strong, hard, stiff
(iii) Skin — elastic (decreases with age), soft, tough
(iv) Plastic — tough, stiff
(v) Rubber — elastic, tough
(vi) Plastic — tough
(vii) Nylon — elastic
(viii) Rubber — tough, elastic
(ix) Plastic — plastic, tough
(x) Putty or silicon — depends on material and purpose