Unit 3 Area of study 1: Motion in one and two dimensions

Chapter 1

Forces in action

1. $u = 0$, $x = 10$ m, $t = 4.0$ s, $a = ?$

$$x = ut + \tfrac{1}{2}at^2$$

$\Rightarrow 10 = \tfrac{1}{2} \times a \times 16$

$\Rightarrow 10 = 8a$

$\Rightarrow a = 1.25 \text{ m s}^{-2}$

Over first 5.0 m

$u = 0$, $x = 5.0$ m, $a = 1.25 \text{ m s}^{-2}$

$v = ?$

$v^2 = u^2 + 2ax$

$= 2 \times 1.25 \times 5.0$

$\Rightarrow v = 3.5 \text{ m s}^{-1}$

2. (a) As they are flying in opposite directions, the speed of bird 1 can be considered negative.

$v_2 = v_2 - v_1$

$= 5 - -5$

$= 10 \text{ m s}^{-1}$

(b) $s = ut$

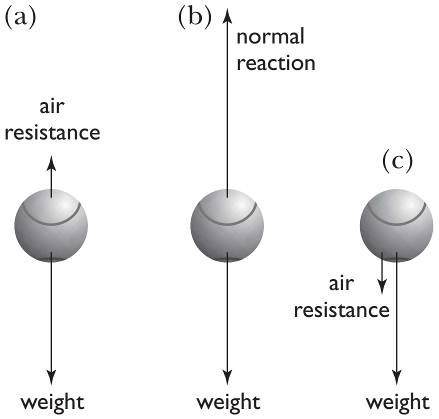
$\therefore t = \dfrac{s}{u}$

$= \frac{15}{10}$

$= 1.5$ s

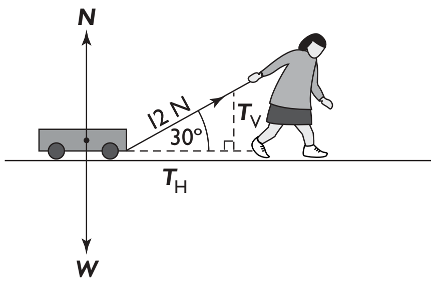
3. 9.8 m s^{-2} down

4. (a) (b) (c)



5. Time and space have the same measurements in all frames of reference.

6.



(a) $\boldsymbol{W} = m\boldsymbol{g}$

$= 4.0 \times 10$

$= 40$ N

(b) $T_H = 12 \cos 30°$

$= 10$ N

(c) Net vertical force = 0

$\Rightarrow N + T \sin 30° = W$

$\Rightarrow N + 12 \sin 30° = 40$

$\Rightarrow N = 40 - 12 \sin 30°$

$= 34$ N

7. (a) F (down)

(b) C (perpendicular to surface)

(c) X (Constant velocity must be the result of a zero net force.)

8. (a) Δp = impulse

= area under graph

$= 10 \text{ N} \times 6 \text{ s}$

$= 60 \text{ kg m s}^{-1}$

(b) Δp = impulse

= area under graph

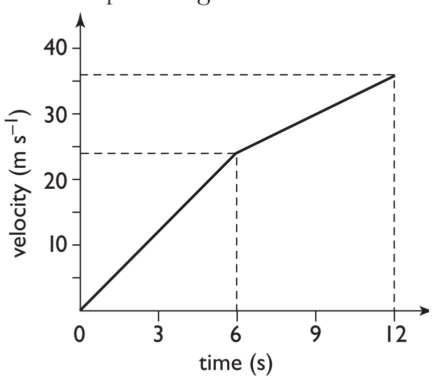
$= 10 \text{ N} \times 6 \text{ s} + 5 \text{ N} \times 6 \text{ s}$

$= 90 \text{ kg m s}^{-1}$

$\Rightarrow P_f - P_i = 90 \text{ kg m s}^{-1}$

$\Rightarrow P_f = 90 \text{ kg m s}^{-1}$

(c)



9. (a) $\boldsymbol{F}_{net} = m\dfrac{\Delta \boldsymbol{v}}{\Delta t}$

$= 0.200 \times \dfrac{-2.7}{0.10}$

= 5.4 N opposite to the original direction of motion

(b) Impulse on billiard ball = $m\Delta \boldsymbol{v}$
$= 0.200 \times -2.7$
$= 0.54$ N s
opposite to the original direction of motion

(c) The table doesn't move because the net force acting on it is zero. It is zero because friction applied to the table by the floor is enough to match the force applied by the ball.

10. (a) The net force on the bullet is large because the forces acting on it, other than the one exerted by the rifle, are small. The net force on the rifle is zero if the shooter exerts a force on it to balance its weight and the force applied by the bullet. Therefore, the rifle does not accelerate.

(b) $P_f = P_i$

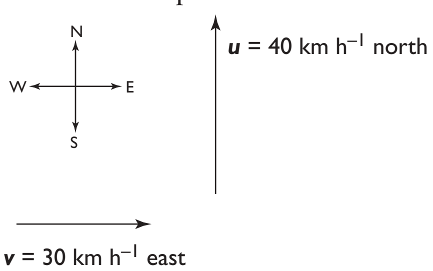
$\Rightarrow 4.0 \text{ kg} \times v_{\text{rifle}} + 0.020 \text{ kg} \times 300 \text{ m s}^{-1} = 0$

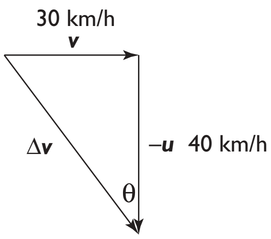
$\Rightarrow 4.0v_{\text{rifle}} = -6.0$

$\Rightarrow v_{\text{rifle}} = -1.5 \text{ m s}^{-1}$

$\Rightarrow$ recoil speed is 1.5 m s^{-1}

11.





(a) $\Delta \boldsymbol{v} = \boldsymbol{v} - \boldsymbol{u}$

$\Rightarrow \Delta v = 50 \text{ km h}^{-1}$

$\tan\theta = \dfrac{30}{40}$

$\Rightarrow \theta = 37°$

$\boldsymbol{a} = \dfrac{\Delta \boldsymbol{v}}{\Delta t}$

$= \dfrac{50 \text{ km h}^{-1} \text{ S } 37°\text{E}}{2.0 \text{ s}}$

$= 25 \text{ km h}^{-1} \text{ s}^{-1} \text{ S } 37°\text{E}$

(b) $a = \dfrac{25 \text{ km h}^{-1}}{1 \text{ s}} \text{ S } 37°\text{E}$

$= \dfrac{(25 \div 3.6) \text{ ms}^{-1}}{1 \text{ s}} \text{ S } 37°\text{E}$

$= 6.9 \text{ m s}^{-2} \text{ S } 37°\text{E}$

12. (a) $a = \dfrac{\Delta v}{t}$

$= \dfrac{-2.0 - 5.0}{0.20}$

$= -35 \text{ m s}^{-2}$

$= 35 \text{ m s}^{-2}$ opposite to the initial direction of the dodgem car.

(b) $F_{\text{net}} = ma$
$= 200 \times 35$
$= 7.0 \times 10^3$ N

13. (a) Braking distance = area under graph over last 20 s

$= \frac{1}{2} \times 20 \text{ s} \times 20 \text{ m s}^{-1}$

$= 200$ m

(b) Total distance travelled by train (and cyclist)
= area under graph

$= \frac{1}{2} \times 50 \text{ s} \times 20 \text{ m s}^{-1} + 50 \text{ s} \times 20 \text{ m s}^{-1} + 200 \text{ m}$

$= 1700$ m

constant speed $= \dfrac{\text{distance travelled}}{\text{time taken}}$

$= \dfrac{1700 \text{ m}}{120 \text{ s}}$

$= 14 \text{ m s}^{-1}$

(c) $a = \dfrac{\Delta v}{\Delta t}$

$= \dfrac{20 \text{ m s}^{-1}}{50 \text{ s}}$

$= 0.4 \text{ m s}^{-2}$

$F_{\text{net}} = ma$
$= 4.0 \times 10^4 \times 0.40$
$= 16\,000$ N

Total frictional forces = 8000 N

$\Rightarrow$ Forward force − 8000 N = 16 000 N

$\Rightarrow$ Forward force = 24 000 N

$= 2.4 \times 10^4$ N

(d) $a = \dfrac{\Delta v}{\Delta t}$

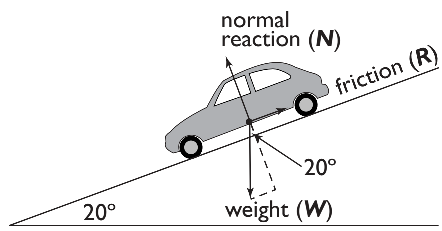
$= \dfrac{-20 \text{ m s}^{-1}}{20 \text{ s}}$

$= -1.0 \text{ m s}^{-2}$

$F_{\text{net}} = ma$
$= -4.0 \times 10^4 \times 1.0$
$= -40\,000$ N

Additional frictional force = 40 000 − 8000
= 32 000 N
$= 3.2 \times 10^4$ N

14. (a)



(b) $N = W\cos 20°$
$= mg\cos 20°$
$= 1500 \times 10 \times \cos 20°$
$= 1.4 \times 10^4$ N

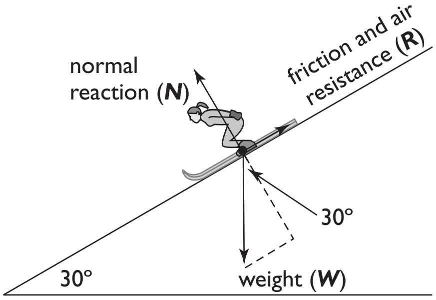
(c) Zero (Car is at rest.)

(d) $F_{\text{net}} = 0$

$\Rightarrow mg\sin 20° - R = 0$

$\Rightarrow R = mg\sin 20°$
$= 1500 \times 10 \times \sin 20°$
$= 5.1 \times 10^3$ N

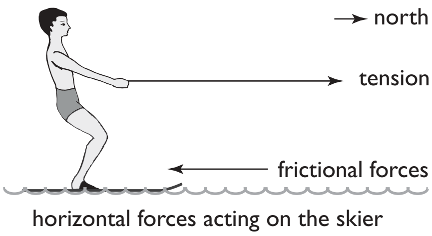
15. (a) No direction. The net force is zero.
(b)

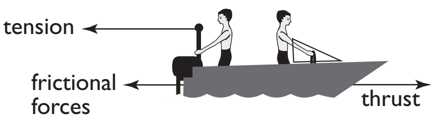


$F_{net} = 0$
$\Rightarrow mg \sin 30° - (\text{air resistance} + \text{friction}) = 0$
$\text{air resistance} + \text{friction} = mg \sin 30°$
$= 60 \times 10 \times \sin 30°$
$= 300$ N

16. The stationary car is pushed forward by the other vehicle. As a result, the seat pushes the body of an occupant forward. This happens almost instantaneously. However, without a headrest, there is nothing to push the occupant's head forward quickly. The head remains at rest until pulled forward by the spine (Newton's First Law of Motion). The head applies an equal and opposite force to the spine, (Newton's Third Law of Motion) potentially causing serious injuries.

17.





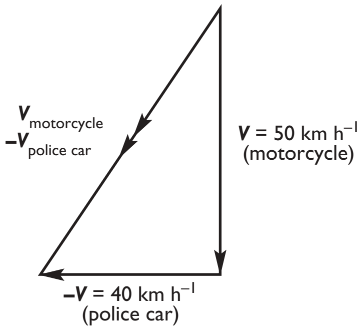
(a) $\boldsymbol{F}_{net} = m\boldsymbol{a}$
$= 70 \times 2.0$
$= 140$ N north
$\Rightarrow$ tension − frictional forces = 140
$\Rightarrow$ tension − 240 = 140
$\Rightarrow$ tension = 380 N north

(b) $\boldsymbol{F}_{net} = m\boldsymbol{a}$
$= 350 \times 2.0$
$= 700$ N north
$\Rightarrow$ thrust − tension − frictional forces = 700
$\Rightarrow$ thrust − 380 − 600 = 700
$\Rightarrow$ thrust = 1680 N north (1.7×10^3 N north)

18. In each of cases (a), (b) and (c), the change in momentum is fixed. An increase in the time interval during which the momentum changes results in a smaller force applied to the occupants or cyclist as $\Delta \boldsymbol{p} = \boldsymbol{F}\Delta t$

19. The car is travelling 18 km h^{-1} faster than the truck so
$V_{car} = 80 + 18$
$= 98$ km h^{-1}

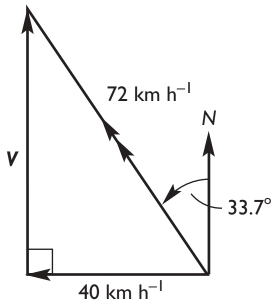
20. (a) $v_{\text{motorcycle rel police car}} = v_{\text{motorcycle}} - v_{\text{police car}}$



$v_{\text{motorcycle rel police car}} = \sqrt{50^2 + 40^2}$
$= 64$ km h^{-1}

(b) 64 km h^{-1}

(c) $v_{\text{limousine rel police car}} = v_{\text{limousine}} - v_{\text{police car}}$



$\tan 33.7° = \frac{40}{v}$
$\Rightarrow v = \frac{40}{\tan 33.7°}$
$= 60$ km h^{-1}

Alternatively, use Pythagoras' theorem.

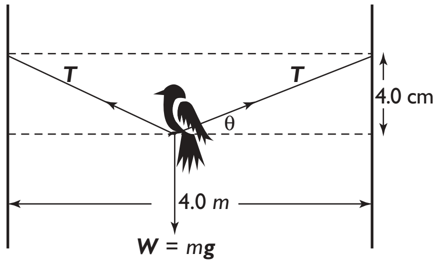
21. (a) Assigning original direction of motion of car as positive:
(i) $\Rightarrow$ Impulse $= m\Delta v$
$= 70\ (0 - 14)$
$= -980$ N s
(ii) Impulse $= m\Delta v$
$= 70\ (0 - 14)$
$= -980$ N s
(iii) $x = 0.50$ m, $u = 14$ m s^{-1},
$v = 0$, $a = ?$
$v^2 = u^2 + 2ax$
$\Rightarrow 0 = 196 + 2a \times 0.50$
$\Rightarrow a = -196$ m s^{-2}
(iv) $x = 0.025$ m, $u = 14$ m s^{-1},
$v = 0$, $a = ?$
$v^2 = u^2 + 2ax$
$\Rightarrow 0 = 196 + 2a \times 0.025$
$\Rightarrow a = \frac{-196}{0.05}$
$= -3920$ m s^{-2}

(b) $g = 10$ m s^{-2}
acceleration of driver $= -196$ m s^{-2}
$= \frac{196}{10}\ g$
$= 19.6\ g$
acceleration of passenger's head $= -3920$ m s^{-2}
$= \frac{3920}{10}\ g$
$= 392\ g$

(c) The seatbelts allow the change in momentum of the occupant to take place over a longer time interval, thus reducing the force applied to the occupant. Without a seatbelt, the occupant continues to move forward, colliding with the interior of the car. This 'secondary' collision causes the change in momentum of the occupant to take place in a much smaller time interval than would be the case with a seatbelt. The force applied to the occupant is therefore much greater.

22. To say that the passenger is thrown forward implies that a force accelerates the passenger. The car slows down rapidly in most collisions as a result of a large external force. The passenger continues to move at the original speed of the car while the car slows down.

23.



The net force on the magpie is zero. Resolving vertically:

$2T \sin \theta = mg$

$\Rightarrow T = \dfrac{mg}{2 \sin \theta}$

$\tan \theta = \dfrac{0.04}{2.0}$

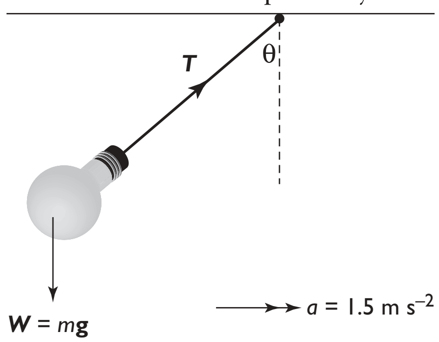
$\Rightarrow \theta = 1.146°$

$\Rightarrow T = \dfrac{4.0 \times 10}{2 \times \sin 1.146°}$

$= 1000$ N

This answer is based on the assumption that the wire has zero mass and is perfectly flexible.

24.



Let the mass of the globe be m and assume that the mass of the wire is negligible. Resolving vertically:

$T \cos \theta = mg$

$\Rightarrow T \cos \theta = 10m$

Resolving horizontally:

$F_{net} = ma$

$\Rightarrow T \sin \theta = ma$

$\Rightarrow T \sin \theta = 1.5m$

Divide equation (2) by equation (1).

$\Rightarrow \dfrac{\sin \theta}{\cos \theta} = \dfrac{1.5}{10}$

$\Rightarrow \tan \theta = 0.15$

$\Rightarrow \theta = 8.5°$

Chapter 2

Collisions

1. No. The system of the two cars is not isolated. There are unbalanced frictional forces acting on the cars during and immediately after the collison.

2. (a) The system of the coal and railway cart can be considered to be isolated if the horizontal motion only is considered.

$P_i = 500 \times 3.0 + 0$

(initial horizontal momentum of the coal is zero)

$= 1500$ kg m s^{-1} south

$P_f = 750\ v$

where v = final velocity of cart

$P_f = P_i$

$\Rightarrow 750\ v = 1500$ south

$\Rightarrow v = 2.0$ m s^{-1} south

(b) The vertically downward momentum of the coal decreases to zero because there is an upward net force acting on it when it strikes the cart. The total momentum of the Earth–coal system has not changed.

(c) $P_i = 750 \times 2.0$

$= 1500$ kg m s^{-1} south

When the coal falls from the cart it has a horizontal velocity of 2.0 m s^{-1} south.

$\Rightarrow 500\ v + 250 \times 2.0 = 1500$ kg m s^{-1} south

$\Rightarrow 500\ v = (1500 - 500)$ kg m s^{-1} south

$\Rightarrow v = \dfrac{1000}{500}$ kg m s^{-1} south

$= 2.0$ m s^{-1} south

3. (a) F_{spring} + weight = 0

$\Rightarrow F_{spring} = -mg$

$= 1.0 \times 10$

$= 10$ N up

(b) $k = \dfrac{F}{x}$

= gradient of F vs x graph

$= \dfrac{15 \text{ N}}{0.40 \text{ m}}$

$= 38$ N m^{-1} (37.5 N m^{-1})

(c) A — the spring with the greatest spring constant

(d) $W = mg$

$= 0.500 \times 10$

$= 5.0$ N

$x = 0.50$ m (from graph)

Work done = area under F vs x graph

$= \frac{1}{2} \times 0.50 \times 5.0$

$= 1.3$ J (1.25 J)

(e) Greatest strain energy occurs in the spring with the F vs x graph of greatest area.

At maximum extension

Area A = $\frac{1}{2} \times 0.20 \times 25 = 2.5$ J

Area B = $\frac{1}{2} \times 0.40 \times 15 = 3.0$ J

Area C = 1.25 J (from (d))

Spring B has greatest strain energy.

4. (a) Energy stored

= energy under force vs compression graph

= area under force vs length graph as length changes from 20 cm to 5 cm

$= \frac{1}{2} \times 0.15 \text{ m} \times 33 \text{ N}$
$= 2.5 \text{ J} \ (2.475 \text{ J})$

(b) $\Delta E_k = 2.4755$
$E_k = 2.475 \text{ J}$ (since initial $E_k = 0$)
$\Rightarrow \frac{1}{2} mv^2 = 2.475 \text{ J}$
$\Rightarrow v = \sqrt{\frac{2.475 \times 2}{2.5}}$
$= 1.4 \text{ m s}^{-1}$

(c) $F = kx$, where x = compression
When $x = 10$ cm, $F = 22$ N
$k = \frac{F}{x}$
$= \frac{22 \text{ N}}{0.10 \text{ m}}$
$= 220 \text{ N m}^{-1}$

5. (a) No. The total kinetic energy of the ball and the ground is less after the collision than it was before the collision.
(b) Sound, along with some heating of the ball, provide evidence that some of the ball's initial kinetic energy is transformed.
(c) Yes, assuming that the ball–ground system is an isolated system

6. (a) $p = mv$
$= 70 \times 2.0$ east
$= 140 \text{ kg m s}^{-1}$ east
(b) Three seconds before impact, Dean is 6 m from Melita because he is gliding at 2.0 m s^{-1} east.
Taking Dean's position as the origin:
$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$
$= \frac{70 \times 0 + 50 \times 6}{120}$
$= 2.5$ m
The centre of mass is 2.5 m east of Dean.
(c) Before the collision, the centre of mass travels 3.5 m (from Dean to Melita) in 3.0 s.
$v = \frac{\Delta x}{\Delta t}$
$= \frac{3.5 \text{ m east}}{3.0 \text{ s}}$
$= 1.2 \text{ m s}^{-1}$ east
(d) The momentum of the centre of mass remains constant.
$\Rightarrow$ The common velocity of Melita and Dean after the collision is 1.2 m s^{-1} east.

7. (a) Assigning east as positive
$p_i = 1500 \times -20 + 2000 \times 20$
$= 10\,000 \text{ kg m s}^{-1}$
$p_f = p_i$
$\Rightarrow 3500v = 10\,000$
$\Rightarrow v = 2.9 \text{ m s}^{-1}$ east (2.86 m s^{-1} east)
(b) $\Delta v_{car} = v - u = 2.9 - (-20)$
$= 22.9 \text{ m s}^{-1}$
$\Delta v_{truck} = v - u = 2.9 - 20$
$= -17.1 \text{ m s}^{-1}$
$\Rightarrow$ The car experiences the greatest (in magnitude) change of velocity.
(c) $\Delta p_{car} = -\Delta p_{truck}$
since the total change in momentum is zero. This can be verified.
$\Delta p_{car} = p_f - p_i$
$= 1500 \times 2.86 - (-30\,000)$
$= 3.4 \times 10^4 \text{ kg m s}^{-1}$
$\Delta p_{truck} = p_f - p_i$
$= 2000 \times 2.86 - 40\,000$
$= -34\,200 \text{ kg m s}^{-1}$
$= -3.4 \times 10^4 \text{ kg m s}^{-1}$
(d) Each vehicle experiences the same force (in magnitude) (Newton's Third Law of Motion).

8. (a) $E_k = \frac{1}{2} mv^2$
$= \frac{1}{2} \times 900 \times 20^2$
$= 1.8 \times 10^5 \text{ J}$
(b) $1.8 \times 10^5 \text{ J}$
(c) $x = 0.40$ m, $u = 20 \text{ m s}^{-1}$, $v = 0 \text{ m s}^{-1}$
$v^2 = u^2 + 2as$
$a = \frac{v^2 - u^2}{2x}$
$= \frac{0 - 20^2}{2 \times 0.4}$
$= \frac{-400}{0.8}$
$= 500 \text{ m s}^{-2}$
$F = ma$
$= 900 \times 500$
$= 4.5 \times 10^5$ N opposite to the initial direction of motion of the car

9. (a) Assume that the sum of the forces other than that of the seatbelt applied to the driver is zero.
$W = \Delta E_k$
$= \frac{1}{2} \times 70 \times \left(\frac{60}{3.6}\right)^2$
$= 9.7 \times 10^3 \text{ J}$
(b) $F_{av}\, x = 9722$
$\Rightarrow F_{av} = \frac{9722}{0.70}$
$= 1.4 \times 10^4$ N
(c) An estimate of the depression of the dashboard by the driver needs to be made. If the dashboard is depressed by 5 mm, then
$F_{av} = \frac{9722}{0.005}$
$= 2 \times 10^6$ N (approx.)

10. (a) $\Delta E_k = \frac{1}{2} mv^2$
$= \frac{1}{2} \times 450 \times (2.0)^2$
$= 900 \text{ J}$
$\Rightarrow$ Maximum strain energy = 900 J
$\Rightarrow \frac{1}{2} kx^2 = 900 \text{ J}$
k = gradient of F vs compression graph
$= \frac{300 \text{ kN}}{0.01 \text{ m}}$
$= 3.0 \times 10^7 \text{ N m}^{-1}$

$\Rightarrow \frac{1}{2} \times 3.0 \times 10^7 \times x^2 = 900$

$$\Rightarrow x = \sqrt{\frac{900 \times 2}{3.0 \times 10^7}}$$
$$= 7.7 \times 10^{-3} \text{ m}$$

(b) Work done = strain energy
= 900 J

(c) 2.0 m s^{-1}. If the rubber bumper obeys Hooke's Law as it expands to its original shape, all of the energy will be returned to the kinetic energy of the dodgem car.

11. (a) Noise, heating and any damage to the car indicate that some of the initial kinetic energy is transformed.
(b) Yes. On an icy road, friction is small enough for the system of the two cars to be considered isolated.

12. (a) Assign east as positive.
$P_i = mu + m \times (-20)$
$P_f = 2m \times 5$
$\Rightarrow mu - 20m = 10m$
$\Rightarrow u - 20 = 10$
$\Rightarrow u = 30 \text{ m s}^{-1}$
$= 30 \text{ m s}^{-1}$ east

(b) $E_{ki} = \frac{1}{2} m(30)^2 + \frac{1}{2} m(20)^2$

$E_{kf} = \frac{1}{2} \times 2m \times (5)^2$

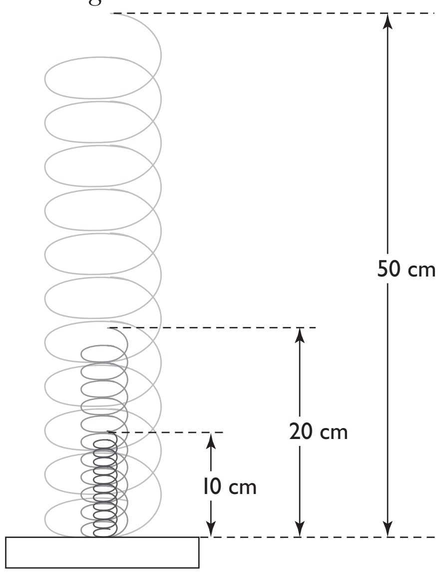
$$\frac{E_{kf}}{E_{ki}} = \frac{m \times 25}{\frac{1}{2} m(900 + 400)}$$
$$= 0.038 \left(\frac{1}{26}\right)$$

13. (a) $P_i = mv - mv$
$= 0$
$\Rightarrow P_f = 0$
$\Rightarrow v = 0$

(b) As the collision is perfectly elastic, the kinetic energy of each of the two cars has not changed. As the force on each car in the collision is equal (by Newton's Third Law) and their masses are equal, they have both accelerated equally in the collision. Their speeds must then be equal and unchanged: that is, 60 km h^{-1}.

14. Student responses will vary. In most situations, an occupant in a larger car is safer. However, the design of the particular car, the nature of the collision and several other factors influence the likely effect of an accident on occupants. The following points should be made.
- The forces applied to each car by the other are equal in magnitude and opposite in direction.
- The change in velocity of each car is dependent on the mass of the car. Assuming that the sum of the forces other than that applied by the other car is zero, the change in velocity is inversely proportional to the mass of the car.
- Assuming that you are properly restrained and that the collision is head-on, your change in velocity (and therefore the deceleration you are subjected to) is less if you are in a heavier car.
- The body continues to move in the original direction and at the original speed of your car until an unbalanced force acts on you. If you are not restrained, the unbalanced force will be provided by the windscreen or part of the interior of the car, which has already slowed down. A smaller car will have slowed down more, so the impulse applied to you ($m\Delta v$) will be greater.

15. (a) The springs are inside each other as shown in the diagram below.



(b) The longest spring is the only one compressed for the first 30 cm of compression.

$k = \text{gradient} = \frac{F}{x} = \frac{20 \text{ N}}{0.30 \text{ m}}$
$= 67 \text{ N m}^{-1}$

(c) When the springs are compressed between 30 cm and 40 cm, the longest spring and the middle spring are compressed. Let the spring constants be k_l and k_m respectively.
$F = k_l x + k_m x$
$\Rightarrow F = (k_l + k_m) x$
gradient $= k_l + k_m$
$= \frac{10 \text{ N}}{0.10 \text{ m}}$
$k_l + k_m = 100 \text{ N m}^{-1}$
When springs are compressed between 40 cm and 50 cm, all three springs are compressed. Let the spring constant of the shortest spring be k_s.
$F = k_l + k_m x + k_s x$
$\Rightarrow F = (k_l + k_m + k_s) x$
gradient $= k_l + k_m + k_s$
$= \frac{20 \text{ N}}{0.10 \text{ m}}$
$\Rightarrow k_l + k_m + k_s = 200 \text{ N m}^{-1}$
but $k_l + k_m = 100 \text{ N m}^{-1}$
$\Rightarrow k_s + 100 = 200$
$\Rightarrow k_s = 100 \text{ N m}^{-1}$

16. (a) Assign north as positive.
$P_i = 0.200 \times 2.0 + 0$
$= 0.400 \text{ kg m s}^{-1}$
$P_f = 0.200 v_w + 0.200 \times 1.7$
$P_f = P_i$
$\Rightarrow 0.200 v_w + 0.34 = 0.400$
$\Rightarrow v_w = 0.30 \text{ m s}^{-1}$
$= 0.30 \text{ m s}^{-1}$ north

(b) $E_{ki} = \frac{1}{2} \times 0.200 \times (2.0)^2 + 0$
$= 0.40$ J
$E_{kf} = \frac{1}{2} \times 0.200 \times (1.7)^2 + \frac{1}{2} \times 0.200 \times (0.30)^2$
$= 0.298$ J

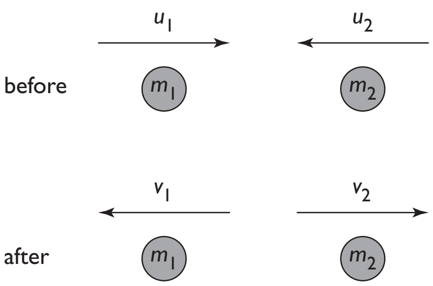
Percentage returned $= \frac{0.298}{0.40} \times \frac{100\%}{1}$
$= 75\%$

(c) (i) $P_i = 0.200 \times 2.0$
$= 0.40$ kg m s^{-1}
$P_f = 0.200 \times -0.5 + 0.200 \times 2.5$
$= 0.40$ kg m s^{-1}
$\Rightarrow$ momentum is conserved.

(ii) The player's claim suggests that the total kinetic energy of balls after the collision is greater than the total kinetic energy of the balls before the collision. This is not possible unless energy is transferred to the system other than by the cue.

$E_{ki} = \frac{1}{2} \times 0.200 \times (2.0)^2$
$= 0.40$ J
$E_{kf} = \frac{1}{2} \times 0.200 \times (0.5)^2 + \frac{1}{2} \times 0.200 \times (2.5)^2$
$= 0.65$ J

17.



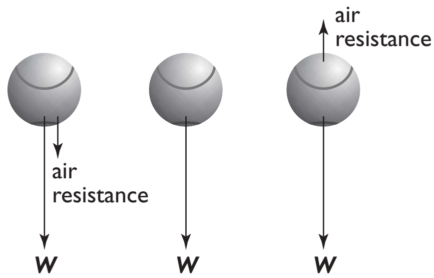
$$P_i = P_f$$
$$\Rightarrow m_1u_1 - m_2u_2 = -m_1v_1 + m_2v_2$$
$$\Rightarrow m_1u_1 + m_1v_1 = m_2v_2 + m_2u_2$$
$$\Rightarrow m_1\ (u_1 + v_1) = m_2\ (v_2 + u_2)$$
$$\Rightarrow \frac{m_1}{m_2} = \frac{v_2 + u_2}{u_1 + v_1} \qquad (1)$$
$$E_{ki} = E_{kf}$$
$$\Rightarrow \tfrac{1}{2} m_1{u_1}^2 + \tfrac{1}{2} m_2{u_2}^2 = \tfrac{1}{2} m_1{v_1}^2 + \tfrac{1}{2} m_2{v_2}^2$$
$$\Rightarrow m_1{u_1}^2 - m_1{v_1}^2 = m_2{v_2}^2 - m_2{u_2}^2$$
$$\Rightarrow m_1\ ({u_1}^2 - {v_1}^2) = m_2\ ({v_2}^2 - {u_2}^2)$$
$$\Rightarrow \frac{m_1}{m_2} = \frac{v_2^2 - u_2^2}{u_1^2 - v_1^2} \qquad (2)$$

Combining (1) and (2)

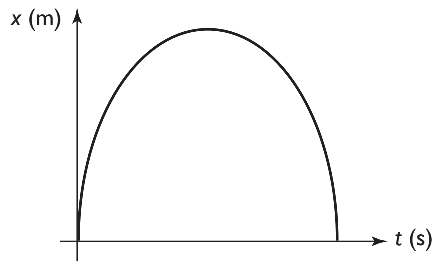
$$\frac{v_2 + u_2}{u_1 + v_1} = \frac{v_2^2 - u_2^2}{u_1^2 - v_1^2}$$
$$\Rightarrow \frac{v_2 + u_2}{u_1 + v_1} = \frac{(v_2 - u_2)(v_2 + u_2)}{(u_1 - v_1)(u_1 + v_1)}$$
$$\Rightarrow u_1 - v_1 = v_2 - u_2$$
$$\Rightarrow u_2 - u_1 = v_2 + v_1$$

Projectile and circular motion

1.

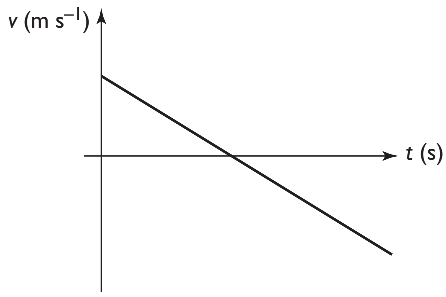


2. (a)



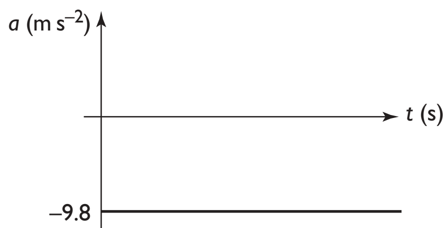
The velocity of the projectile at any point is found by the gradient of the *x* vs *t* graph.

(b)



The acceleration of the projectile at any point is found by the gradient of the *v* vs *t* graph.

(c)



This graph shows the velocity is constant at –9.8 m s^{-1}. The negative sign indicates the acceleration is towards the Earth.

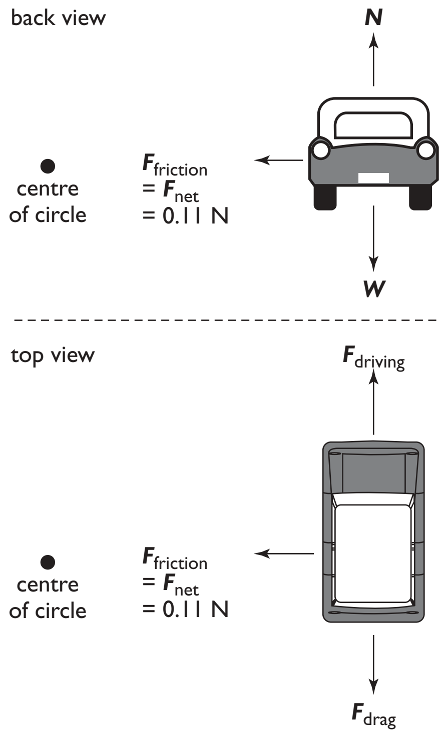
3. (a) Vertical component:
$v_v = 20 \sin 50° = 15$ m s^{-1}
Horizontal component:
$v_h = 20 \cos 50° = 13$ m s^{-1}

(b) Vertical component:
$v_v = 11 \cos 23° = 10$ m s^{-1}
Horizontal component:
$v_h = 11 \sin 23° = 4.3$ m s^{-1}

(c) Vertical component:
$v_v = 5$ m s^{-1}
Horizontal component:
$v_h = 5 \sin 0° = 0$ m s^{-1}

(d) Vertical component:
$v_v = 10 \sin 0° = 0$ km h^{-1}
Horizontal component:
$v_h = 10$ km h^{-1}

(e) Vertical component:
$v_v = 33 \cos 60° \text{ or } 33 \sin 30° = 17 \text{ m s}^{-1}$
Horizontal component:
$v_h = 33 \sin 60° \text{ or } 33 \cos 30° = 29 \text{ m s}^{-1}$

4. As no forces are acting in the horizontal direction, there can be no horizontal acceleration. Therefore, the horizontal component of velocity must remain constant.
5. The time of a projectile's flight is the time it takes to hit the ground. Therefore, the projectile cannot take longer to complete one part of its motion than the other. Time is the only useful variable that is a scalar and is the same in both the vertical and horizontal directions.
6. (a) $r = 120$ m
$v = 6.0 \text{ km h}^{-1}$
$= 1.7 \text{ m s}^{-1}$
$a = ?$
$$a = \frac{v^2}{r} = \frac{(1.7)^2}{120}$$
$a = 0.02 \text{ m s}^{-2}$ (0.024)
towards the centre of the circle
(b) $a = 0.024 \text{ m s}^{-2}$
$m = 65$ kg
$F = ?$
$F_c = F_{net} = ma$
$= 65 \times 0.024$
$= 1.6$ N
towards the centre of the circle
7. $T = 35$ s
$r_N = 2.5$ m
$r_L = 3.2$ m
$$a_N = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2(2.5)}{(35)^2} = 0.08 \text{ m s}^{-2}$$
$$a_L = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2(3.2)}{(35)^2} = 0.10 \text{ m s}^{-2}$$
∴ Lucy experiences the greatest centripetal acceleration.
8. $r = 350$ m
$$v = 15 \text{ km h}^{-1} \times \frac{1000}{3600} = 4.2 \text{ m s}^{-1}$$
(a) $$a = \frac{v^2}{r} = \frac{(4.2)^2}{350} = 0.050 \text{ m s}^{-2}$$
towards the centre of the circle
(b) $m = 35$ kg
$a = 0.05 \text{ m s}^{-2}$
$F_{net} = F_c = ma$
$= 35 \times 0.05$
$= 1.75$ N
towards the centre of the circle
(1.7 N if using more than 2 sig. fig. for a)
(c) $m = 1500$ kg
$a = 0.05 \text{ m s}^{-2}$
$F_{net} = F_c = ma$
$= 1500 \times 0.05$
$= 75$ N
towards the centre of the circle
(d) To move along the same path, the child and the train require the same *acceleration*. As the mass of the child and the train are different, different forces are needed to produce identical accelerations.
9. (a) $r = 65 \text{ cm} = 0.65$ m
$m = 0.12$ kg
$T = 5.2$ s
$a = ?$
$$a = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2(0.65)}{(5.2)^2} = 0.95 \text{ m s}^{-2}$$
towards the centre of the circle
(b) $a = 0.95 \text{ m s}^{-2}$
$m = 0.120$ kg
$F = ?$
$F_{net} = F_c = ma$
$= 0.12 \times 0.95$
$= 0.11$ N
towards the centre of the circle
(c)


10. Newton's first law states that an object will continue to move in a straight line with constant speed unless an unbalanced force acts on it. Therefore, the mass will continue to move forwards without a propelling force, once in motion. The centripetal force acts to change the direction of the mass, not its speed.
11. To go around a bend, a motorcyclist needs a horizontal force acting on the bike towards the centre of the curve of the bend. This is provided by the road acting on the tyres. The force of the road on the tyres needs to act through the centre of mass of the cyclist, otherwise the force will act to tip the bike

over. As the horizontal component of this force is acting towards the centre of the curve, the motor-cyclist must lean into the curve to avoid falling off.

12. (a) $m = 10$ kg
$g = 9.70$ N kg^{-1}
$W = ?$
$W = mg$
$= 10 \times 9.70$
$= 97$ N

(b) Consider up to be positive.
$\boldsymbol{F}_{net} = \boldsymbol{W} + \boldsymbol{F}_{up}$
$\Rightarrow 0 = \boldsymbol{W} + \boldsymbol{F}_{up}$
$\Rightarrow \boldsymbol{F}_{up} = -\boldsymbol{W}$
$= -(-97 \text{ N})$
$= 97$ N

(c) $g = 9.70$ N kg^{-1}
$r = ?$
$M_{earth} = 5.98 \times 10^{24}$ kg
$\mathrm{G} = 6.67 \times 10^{-11}$ N m^2 kg^{-2}

$$g = \frac{\mathrm{G}M}{r^2}$$

$$r = \sqrt{\frac{\mathrm{G}M}{g}}$$

$$r = \sqrt{\frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{9.70}}$$

$r = 6.41 \times 10^6$ m

13. Gravity holds solar systems together. As gravity is the only force acting on a planet, it is the net force. It is directed toward the sun. Whenever the net force on an object is toward the centre of a circle, the object will experience centripetal motion.

14. According to Newton's first law, an object will move in a straight line, with constant speed, unless an unbalanced force is acting on it. As gravity acts at right angles to the satellite's velocity, it does not change the speed of the satellite; rather it changes its direction. This causes the satellite to move around the Earth. Because the force of gravity and the speed of the satellite remain constant, so must its radius as $r = \frac{mv^2}{F}$; therefore it cannot move closer to the Earth.

15. As $W = Fx$, and $g = \frac{F}{m}$:

Area under a g vs x graph has the same units as $\frac{F}{m}x = \frac{W}{m}$ (i.e. energy per kg).

16. $x = 150$ m (consider down as positive)
$u = 0$ m s^{-1}
$a = 10$ m s^{-2} (due to gravity)

(a) $t = ?$

$$x = ut + \tfrac{1}{2}at^2$$

$$150 = 0t + \tfrac{1}{2}(10)t^2$$

$150 = 5t^2$
$30 = t^2$
$\Rightarrow t = \sqrt{30} = 5.5$ s
(take the positive square root)

(b) $v = ?$
$v^2 = u^2 + 2ax$
$v^2 = 0^2 + 2(10)(150)$
$v^2 = 3000$
$v = \sqrt{3000} = 55$ m s^{-1}

17. $u = 18$ m s^{-1}
$a = -10$ m s^{-2} (consider up as positive)

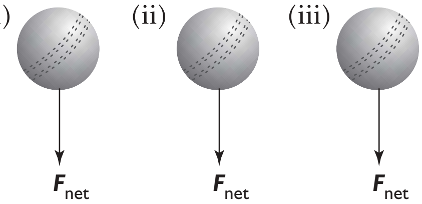
(a) Method 1: (consider whole motion)
$v = -18$ m s^{-1} (due to symmetry)
$t = ?$
$v = u + at$
$-18 = 18 - 10t$
$10t = 36$
$t = 3.6$ s

Method 2: (consider half motion to top of flight)
$v = 0$
$t = ?$
$v = u + at$
$0 = 18 - 10t$
$10t = 18$
$t = 1.8$ s
$\therefore$ for whole motion
$t = 1.8 \times 2$
$t = 3.6$ s

(b) Consider 1st half of motion:
$v = 0$ m s^{-1}
$x = ?$
$v^2 = u^2 + 2ax$
$0^2 = 18^2 + 2(-10)x$
$20x = 18^2$

$$x = \frac{324}{20} = 16 \text{ m}$$

(c) (i) (ii) (iii)



18. $m = 500$ kg
$x = 10$ m
$a = 10$ m s^{-2}
$u = 0$ m s^{-1}
(consider down as positive)

(a) $v = ?$
$v^2 = u^2 + 2ax$
$v^2 = 0^2 + 2(10)(10)$
$v^2 = 200$
$v = 14$ m s^{-1}

(b) $t = ?$

$$x = ut + \tfrac{1}{2}at^2$$

$$10 = 0t + \tfrac{1}{2}(10)t^2$$

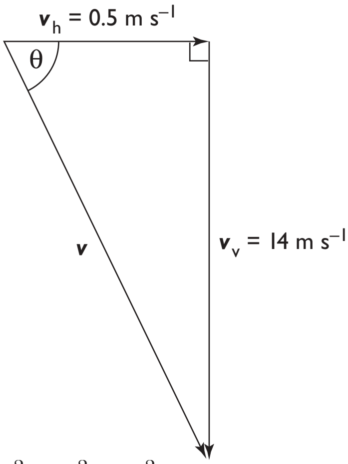
$10 = 5t^2$

$$t^2 = \frac{10}{5} = 2$$

$t = 1.4$ s

(c) horizontally:
$u = 0.5$ m s^{-1}
$a = 0$ m s^{-2} (no forces in this direction)
$t = 1.4$ s (linking factor between horizontal and vertical components)
$x = ?$
$x = ut + \frac{1}{2}at^2$
$x = 0.5(1.4) + \frac{1}{2}(0)(1.4)^2$
$x = 0.7$ m

(d)



$v^2 = v_v^{\,2} + v_h^{\,2}$
$v^2 = 14^2 + 0.5^2$
$v = \sqrt{14^2 + 0.5^2}$
$v = 14$ m s^{-1}
$\tan\theta = \frac{14}{0.5}$
$\Rightarrow \theta = 88°$

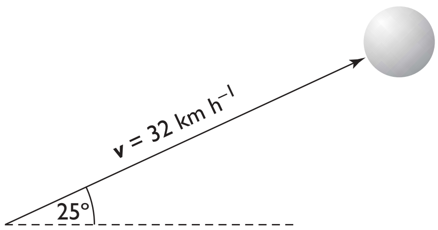
(e) (i) While attached to magnet, $\boldsymbol{a} = 0$ m s^{-2}.
$\Rightarrow \boldsymbol{F}_{net} = m\boldsymbol{a}$
$\boldsymbol{F}_{net} = 500 \times 0$
$\boldsymbol{F}_{net} = 0$ N
(ii) While falling, $\boldsymbol{a} = 10$ m s^{-2} downwards.
$\Rightarrow \boldsymbol{F}_{net} = m\boldsymbol{a}$
$\boldsymbol{F}_{net} = 500 \times 10$
$\boldsymbol{F}_{net} = 5000$ N downwards

19. (a) The car stopped moving because the brakes caused a force to be exerted on the car which opposed its motion. However, no such force was exerted on the tissue box, so it continued to move in a straight line with constant speed as stated in Newton's first law.
(b) $u = 100$ km h^{-1}
(as car is travelling at the same speed)
$= 100 \times \frac{1000}{3600}$ m s^{-1}
$= 28$ m s^{-1}
(c)

Vertical	Horizontal
$u = 0$	$u = 28$ m s^{-1}
$a = 10$ m s^{-2}	$x = 2.5$ m s^{-1}
$x = ?$	

Must find time, as it is the linking factor. Using horizontal component:
$t = \frac{x}{u}$
$t = \frac{2.5}{28}$
$t = 8.9 \times 10^{-2}$ s
(or 9×10^{-2} s if used exact value of u)
Using this with vertical information:
$x = ut + \frac{1}{2}at^2$
$x = 0(8.9 \times 10^{-2}) + \frac{1}{2}(10)(8.9 \times 10^{-2})^2$
$x = 0.040$ m (or 0.041 m if using $t = 9 \times 10^{-2}$ s).
(d) During sudden accelerations, objects could become projectiles, moving through the interior of the car, which could injure the occupants of the car.

20. (a)



$v = 32$ km h^{-1}
$= 32 \times \frac{1000}{3600}$ m s^{-1}
$= 8.9$ m s^{-1}
$v_h = 8.9 \cos 25° = 8.1$ m s^{-1}
$v_v = 8.9 \sin 25° = 3.8$ m s^{-1}
(b) Using vertical components:
$u = 3.8$ m s^{-1}
$a = -10$ m s^{-2}
$v = 0$ m s^{-1}
(considering 1st half of motion only)
$t = ?$
$v = u + at$
$0 = 3.8 - 10t$
$10t = 3.8$
$t = 0.38$ s
$\therefore$ total time $= 0.38 \times 2 = 0.76$ s
(c) Range is horizontal distance, so use horizontal components.
$t = 0.76$ s (as time is the linking factor)
$u = 8.1$ m s^{-1}
$a = 0$ m s^{-2}
$x = ?$
$x = ut$
$= 8.1 \times 0.76$
$= 6.2$ m

21.

Vertical	Horizontal
$u = 7.0 \sin 45°$	$u = 7.0 \cos 45°$
$= 4.9$ m s^{-1}	$= 4.9$ m s^{-1}
$a = -10$ m s^{-2}	$t = 0.98$ s
$v = -4.9$ m s^{-1}	(from vertical)
(due to symmetry)	$x = ?$
$t = ?$	
	$x = ut$
	$x = 4.9 \times 0.98$
$v = u + at$	$x = 4.8$ m
$-4.9 = 4.9 - 10t$	(or 4.9 if maintaining more
$10t = 9.8$	than 2 sig. fig. through
$t = 0.98$ s	working)
	Set up means you will be at a distance of $0.5 \times 11 = 5.5$ m ⇒ your friend will not make the jump and you will be squashed!

22. Let v = initial velocity and θ be the angle of projection.
Then range = $v \cos\theta \times t$.
The time is given by the vertical components,
$v_v = -v \sin\theta$, $u_v = v \sin\theta$, $a = -g$.

$$v = u + at$$
$$-v \sin\theta = v \sin\theta - gt$$
$$t = \frac{2\ v \sin\theta}{g}$$

Now range $= v \cos\theta \times \dfrac{2\ v \sin\theta}{g}$

$$= \frac{2v^2}{g} \sin\theta \cos\theta$$

but $2 \sin\theta \cos\theta = \sin 2\theta$ so

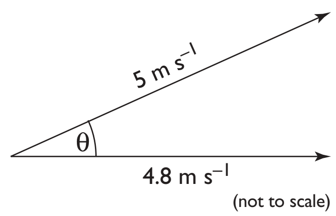
$$\text{range} = \frac{v^2}{g} \sin 2\theta$$

v and g are constant and $\sin 2\theta$ is a maximum when $2\theta = 90°$. So the maximum range is given when $\theta = 45°$.

23. (a) $x = 2$ m
$t = 0.42$ s
$v = ?$

$$v = \frac{x}{t} = \frac{2.0}{0.42} = 4.8 \text{ m s}^{-1} \ (4.76)$$

(b)



$$\cos\theta = \frac{4.76}{5}$$
$$\Rightarrow \theta = 18°$$

(c) Vertical: (consider up as positive)
$u = 5 \sin 18° = 1.545$ m s^{-1}
$a = -10$ m s^{-2}
$v = 0$ m s^{-1}
$x = ?$
$v^2 = u^2 + 2ax$
$0 = (1.545)^2 + 2(-10)x$
$x = 0.12$ m

24. (a) *Vertical component:*
$u = 9.8 \sin 45°$
$= 6.9$ m s^{-1}
$a = -10$ m s^{-1}
$v = 0$ m s^{-1} (at top of flight)
$t = ?$
$v = u + at$
$0 = 6.9 - 10t$
$10t = 6.9$
$t = 0.69$ s

(b) *Vertically:* $x = ?$
$v^2 = u^2 + 2ax$
$0^2 = 6.9^2 + 2(-10)x$
$20x = 6.9^2$
$x = 2.4$ m

Horizontally: $x = ?$
$u = 9.8 \cos 45°$
$= 6.9$ m s^{-1}
$x = ut$
$x = 6.9 \times 0.69$
$x = 4.8$ m

(c) *Horizontal component:*
$x_{\text{top to goal}} = 7.0 \text{ m} - 4.8 \text{ m}$
$= 2.2$ m
$u = 6.9$ m s^{-1}
$t = ?$

$$t = \frac{x}{u}$$
$$t = \frac{2.2}{6.9}$$

$t = 0.32$ s

(d) *Vertical component:*
$u = 0$ m s^{-1}
$t = 0.32$ s
$a = 10$ m s^{-2}
$x = ?$

$$x = ut + \tfrac{1}{2}at^2$$
$$x = 0(0.32) + \tfrac{1}{2}(10)(0.32)^2$$

$x = 0.51$ m

⇒ Final height $= 2.4 - 0.51$
$= 1.9$ m

Therefore the ball goes into the net.

25. (a) *Vertical component:*
$u = 50 \sin 35°$ km h^{-1}

$$= 29 \times \frac{1000}{3600} = 8.0 \text{ m s}^{-1}$$

$a = -10$ m s^{-2}
$v = 0$ m s^{-1}
$t = ?$
$v = u + at$
$0 = 8.0 - 10t$
$t = 0.80$ s

(b) *Vertically:*

$x = ?$
$v^2 = u^2 + 2ax$
$0^2 = 8.0^2 + 2(-10)x$
$20x = 8.0^2$
$x = 3.2 \text{ m}$

Horizontally:

$x = ?$
$u = 50 \cos 35° \times \frac{1000}{3600}$
$= 11 \text{ m s}^{-1}$
$t = 0.80 \text{ s}$
$x = ut$
$x = 11 \times 0.80$
$x = 8.8 \text{ m}$

(equals 9.1 m if use more than 2 sig. fig. in calculations)

(c) *Vertical component:*

$x = 3.2 + 0.8 = 4.0 \text{ m}$
$a = 10 \text{ m s}^{-2}$
$u = 0 \text{ m s}^{-1}$
$t = ?$
$x = ut + \frac{1}{2}at^2$
$4.0 = 0t + \frac{1}{2}(10)t^2$
$4.0 = 5t^2$
$t = 0.89 \text{ s}$

(0.90 s if use more than 2 sig. fig. for x)

(d) *Horizontal component:*

$u = 11 \text{ m s}^{-1}$
$t_{total} = 0.80 + 0.89$
$= 1.7 \text{ s}$
$x = ?$
$x = ut$
$x = 11 \times 1.7$
$x = 19 \text{ m}$

26. (a)

Vertical	Horizontal
$x = -1.7 \text{ m}$	$u = 13.9 \cos 30°$
$a = -10 \text{ m s}^{-2}$	$= 12 \text{ m s}^{-1}$
$u = 13.9 \sin 30°$	$t = 1.6 \text{ s}$
$= 6.95 \text{ m s}^{-1}$	$x = ut = 12 \times 1.6$
$t = ?$	$= 19.2 \text{ m}$
$x = ut + \frac{1}{2}at^2$	Range is 19 m.

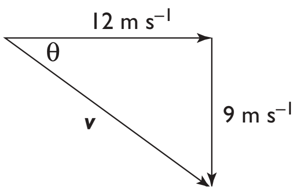
$-1.7 = (6.95)t + \frac{1}{2}(-10)t^2$

$$t = \frac{-6.95 \pm \sqrt{(6.95)^2 - 4(-5)(1.7)}}{2(-5)}$$

$= 1.6 \text{ s}$

(b) $v = ?$
$v = u + at$
$= 6.95 - 10 \times 1.6$
$= -9.05 \text{ m s}^{-1}$

$\tan \theta = \frac{9}{12}$
$\Rightarrow \theta = 37°$
$v = \sqrt{12^2 + 9^2}$
$\therefore v = 15 \text{ m s}^{-1}$



The jumper hits the water with a velocity of 15 m s^{-1} at an angle of 37° to the horizontal.

27. (a) $m = 90 \text{ kg}$

$v = 15 \text{ km h}^{-1} \times \frac{1000}{3600}$
$= 4.2 \text{ m s}^{-1}$
$r = 4.5 \text{ m}$
$F = \frac{mv^2}{r}$
$= \frac{(90)(4.2)^2}{4.5}$
$= 3.5 \times 10^2 \text{ N}$

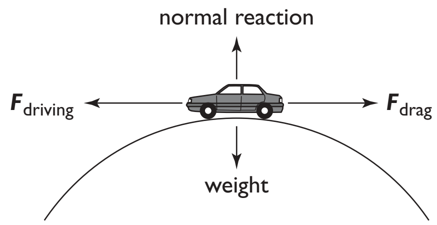
toward the centre of the circle

(b) 3.5×10^2 N toward the centre of the circle (as frictional forces are causing centripetal motion)

(c) $v = 4.2 \text{ m s}^{-1}$
$F = 350 \times 90\%$
$= 315 \text{ N}$
$r = ?$
$F = \frac{mv^2}{r}$
$315 = \frac{90(4.2)^2}{r}$
$r = \frac{90(4.2)^2}{315}$
$r = 5 \text{ m}$

The radius will increase to 5.0 m.

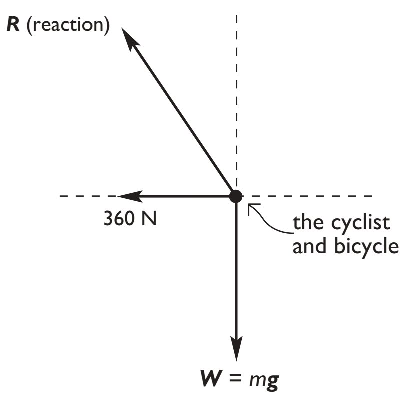
28. (a)



(b) (i) $F_c = F_{net} = W - N$
As N = 0:
$F_c = W = mg$
$F_c = 800 \times 10$
$F_c = 8 \times 10^3$ N downwards

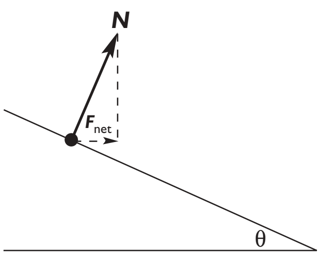
(ii) $F_c = \frac{mv^2}{r}$
$\Rightarrow v^2 = \frac{F_c r}{m}$
$v = \sqrt{\frac{8 \times 10^3 \times 4.0}{800}}$
$v = 6.3 \text{ m s}^{-1}$

29.



$$F_{net} = \frac{mv^2}{r}$$
$$F_{net} = 360 + R \sin 20°$$
$$\Rightarrow \frac{mv^2}{r} = 360 + R \sin 20°$$
but $R \cos 20° = mg$
$$\Rightarrow R = \frac{10m}{\cos 20°}$$
$$\Rightarrow \frac{mv^2}{r} = 360 + \frac{10m \sin 20°}{\cos 20°}$$
$$= 360 + 10m \tan 20°$$
$$\Rightarrow \frac{m(9.0)^2}{10} = 360 + 3.64m$$
$$\Rightarrow 8.1m = 360 + 3.64m$$
$$\Rightarrow 4.46\ m = 360$$
$$\Rightarrow m = 81 \text{ kg}$$
The mass of the bicycle is 20 kg.
∴ The mass of the cyclist is 61 kg.

30. $v = 30 \text{ m s}^{-1}$, $r = 12$ m
$$a = \frac{v^2}{r}$$
$$= \frac{30^2}{12}$$
$$= 75 \text{ m s}^{-2}$$
The horizontal component of the normal reaction force must therefore provide a centripetal acceleration of 75 m s^{-2}.
$N \sin \theta = 75$ m



But $N \cos \theta = mg$
So by dividing, $\tan \theta = \frac{75}{9}$
$$\Rightarrow \theta = 82°$$
The problem with this road is that you would have to drive at 30 m s^{-1} to avoid slipping off!

31. (a) $M_{earth} = 5.98 \times 10^{24}$ kg
$r_{earth} = 6.38 \times 10^6$ m
$m_{person} = 70$ kg
$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}
$g = ?$
$W = ?$
$$g = \frac{GM}{r^2}$$
$$= \frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{(6.38 \times 10^6)^2}$$
$$= 9.80 \text{ N kg}^{-1}$$
$$W = mg = 70 \times 9.80$$
$$= 6.9 \times 10^2 \text{ N}$$

(b) $M_{Mars} = 6.42 \times 10^{23}$ kg
$r_{Mars} = 3.40 \times 10^6$ m
$$g = \frac{GM}{r^2}$$
$g = 3.70$ N kg^{-1}
$W = mg$
$W = 2.6 \times 10^2$ N

(c) $M_{Venus} = 4.87 \times 10^{24}$ kg
$r_{Venus} = 6.05 \times 10^6$ m
$$g = \frac{GM}{r^2}$$
$g = 8.87$ N kg^{-1}
$W = mg$
$W = 6.2 \times 10^2$ N

(d) $M_{Pluto} = 1 \times 10^{22}$ kg
$r_{Pluto} = 1 \times 10^6$ m
$$g = \frac{GM}{r^2}$$
$g = 0.7$ N kg^{-1}
$W = mg$
$W = 5 \times 10^1$ N

32. $r = 5.0 \times 10^5$ m
$g = 4.3$ N kg^{-1}
$m = ?$
$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}
$$g = \frac{GM}{r^2}$$
$$\Rightarrow M = \frac{gr^2}{G}$$
$$M = \frac{4.3(5.0 \times 10^5)^2}{6.67 \times 10^{-11}}$$
$$M = 1.6 \times 10^{22} \text{ kg}$$

33. $M_{sun} = 1.98 \times 10^{30}$ kg
$M_{Earth} = 5.98 \times 10^{24}$ kg
$r_{Earth's\ orbit} = 1.50 \times 10^{11}$ m
$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}
$F = ?$
$$F = \frac{GMm}{r^2}$$
$$= \frac{6.67 \times 10^{-11} \times 1.98 \times 10^{30} \times 5.98 \times 10^{24}}{(1.50 \times 10^{11})^2}$$
$$= 3.5 \times 10^{22} \text{ N}$$

34. $r = r_{Earth} + h$
$= 6.38 \times 10^6 + 3.55 \times 10^5$ m
$= 6.74 \times 10^6$ m
$T = 91$ min
$= 5460$ s
(a) $a = ?$
$$a = \frac{4\pi^2 r}{T^2}$$
$$= \frac{4\pi^2(6.74 \times 10^6)}{(5520)^2}$$
$$= 8.73 \text{ m s}^{-2}$$

(b) $g = ?$

$$g = \frac{GM}{r^2}$$

$$= \frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{(6.74 \times 10^6)^2}$$

$= 8.78\ \text{N kg}^{-1}$

(c) (i) The centripetal acceleration of the space station is caused by acceleration due to gravity (i.e. g). Therefore, the two answers should be the same.

(ii) Discrepancies between the numbers are due to the rounding off of data.

(d) $M_{ss} = 1200$ tonnes

$= 1.2 \times 10^6$ kg

$g = 8.78\ \text{N kg}^{-1}$

$W = ?$

$W = mg$

$= 1.2 \times 10^6 \times 8.78$

$= 1.1 \times 10^7$ N

(e) $r = 10$ m

$M_{astro} = 270$ kg

$M_{ss} = 1.2 \times 10^6$ kg

$G = 6.67 \times 10^{-11}\ \text{N m}^2\ \text{kg}^{-2}$

$F = ?$

$$F = \frac{GMm}{r^2}$$

$$= \frac{6.67 \times 10^{-11} \times 1.2 \times 10^6 \times 270}{(10)^2}$$

$= 2.2 \times 10^{-4}$ N

35. $M_{sun} = 1.98 \times 10^{30}$ kg

$T_{Venus} = 1.94 \times 10^7$ s

$T_{Saturn} = 9.30 \times 10^8$ s

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$$

$$\Rightarrow r = \sqrt[3]{\frac{GMT^2}{4\pi^2}}$$

$$\therefore \frac{r_{Saturn}}{r_{Venus}} = \left(\frac{T_{Saturn}}{T_{Venus}}\right)^{\frac{2}{3}} = \left(\frac{9.30 \times 10^8}{1.94 \times 10^7}\right)^{\frac{2}{3}}$$

$= 13$

36. (a) Work/kg = area under graph from 400 km to 600 km

$\approx \frac{1}{2}(8.7 + 8.2)\ 2 \times 10^5$ (distance in metres)

$= 1.7 \times 10^6\ \text{J kg}^{-1}$

$\Rightarrow$ Work $= 1.7 \times 10^6 \times 800$

$= 1.4 \times 10^9$ J (approx.)

(b) $r = 6.0 \times 10^5 + 6.38 \times 10^6$

$= 6.98 \times 10^6$ m

$M_{Earth} = 5.98 \times 10^{24}$ kg

$T = ?$

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$$

$$T = \sqrt{\frac{r^3 4\pi^2}{GM}}$$

$$T = \sqrt{\frac{(6.98 \times 10^6)^3 4\pi^2}{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}}$$

$T = 5.8 \times 10^3$ s

(c) $\frac{r^3}{T^2}$ = constant for any satellite of the Earth.

(d) (a) would be halved and (b) would remain the same.

37. (a) $F = \frac{GMm}{r^2}$, $G = 6.67 \times 10^{-11}\ \text{N m}^2\ \text{kg}^{-2}$,

$M_{Earth} = 5.98\ 10^{24}$ kg, $M_{satellite} = 2400$ kg

$r = 6.38 \times 10^6 + 2 \times 10^6$

$= 8.38 \times 10^6$

$$F = \frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24} \times 2400}{(8.38 \times 10^6)^2}$$

$= 1.4 \times 10^4$ N

Answers will vary if $F = mg$ is used, taking the value of g from the graph.

(b) Loss of GPE = area under graph from 8.38×10^6 m to 7.18×10^6 m.

This is approximately a trapezium

$A = \frac{1}{2}(a + b)\ h$

$= \frac{1}{2}(5.25 + 6.75) \times 1.2 \times 10^6$

$= 1.7 \times 10^{10}$ J.

(c) It has gained 1.7×10^{10} J of kinetic energy.

Initially $E_K = \frac{1}{2}mv^2$

$= \frac{1}{2} \times 2400 \times 6900^2$

$= 5.7 \times 10^{10}$

After falling $E_K = 5.7 \times 10^{10} + 1.7 \times 10^{10}$

$= 7.4 \times 10^{10}$ J

$\frac{1}{2}mv^2 = 7.4 \times 10^{10}$

$\frac{1}{2} \times 2400 \times v^2 = 7.4 \times 10^{10}$

$v = 7.9 \times 10^3\ \text{m s}^{-1}$

38. $M_{Earth} = 5.98 \times 10^{24}$ kg

$T = 24$ h

$= 8.64 \times 10^4$ s

$r = ?$

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$$

$$r = \sqrt[3]{\frac{GMT^2}{4\pi^2}}$$

$$r = \sqrt[3]{\frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24} \times (8.64 \times 10^4)^2}{4\pi^2}}$$

$r = 4.2 \times 10^7$ m

39.

Vertical	Horizontal
$u = u \sin 28°$ $a = -10\ \text{m s}^{-2}$ $t = \frac{2.5}{2u \cos 28°}$ $v = 0\ \text{m s}^{-1}$	$x = 2.5$ m $u = u \cos 28°$ $t = ?$ $t = \frac{x}{u} = \frac{2.5}{2u \cos 28°}$

Apply the equation $v = u + at$ to the vertical component of the first half of the gymnast's motion.

$$0 = u \sin 28^\circ + (-10)\left(\frac{2.5}{2u \cos 28^\circ}\right)$$

$$0 = u \sin 28^\circ - \left(\frac{25}{2u \cos 28^\circ}\right)$$

$$\left(\frac{25}{2u \cos 28^\circ}\right) = u \sin 28^\circ$$

$$25 = 2u^2 \sin 28^\circ \cos 28^\circ$$

$$u = \sqrt{\frac{12.5}{\sin 28^\circ \cos 28^\circ}}$$

$$u = 5.5 \text{ m s}^{-1}$$

40.

Vertical	Horizontal
$u = 7 \sin \theta$ m s^{-1}	$x = 3.0$ m
$a = -10$ m s^{-2}	$u = 7 \cos \theta$ m s^{-1}
$t = \dfrac{3.0}{14 \cos \theta}$	$t = ?$
$v = 0$ m s^{-1}	$t = \dfrac{x}{u} = \dfrac{3.0}{7 \cos \theta}$

where θ = angle to horizontal

$$v = u + at$$

$$0 = 7 \sin \theta + (-10)\left(\frac{3.0}{14 \cos \theta}\right)$$

$$0 = 7 \sin \theta - \frac{30}{14 \cos \theta}$$

$$\frac{30}{14 \cos \theta} = 7 \sin \theta$$

$$\frac{30}{14 \times 7} = \sin \theta \cos \theta$$

$$0.31 = \sin \theta \cos \theta$$

$$\Rightarrow 2 \sin \theta \cos \theta = 0.62$$

$$\Rightarrow \sin 2\theta = 0.62$$

$$\theta = 19^\circ$$

41. $T = 24$ h

$= 8.64 \times 10^4$ s

$r = 6.38 \times 10^6$ m

$a = ?$

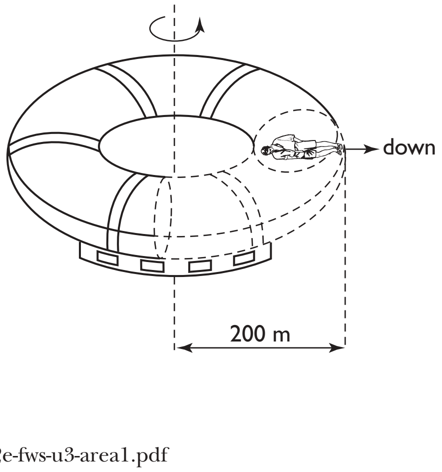
$$a = \frac{4\pi^2 r}{T^2}$$

$$= \frac{4\pi^2(6.38 \times 10^6)}{(8.64 \times 10^4)^2}$$

$$= 0.034 \text{ m s}^{-2}$$

In Victoria, the radius of the circular path would be smaller. If r decreases, acceleration also decreases. Therefore, in Victoria, we experience less centripetal acceleration due to the Earth's motion than people on the Equator.

42. One possible answer.



$a = 10$ m s^{-2}

$r = 200$ m

$T = ?$

$$a = \frac{4\pi^2 r}{T^2}$$

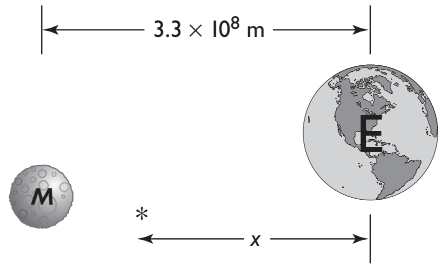
$$T = \sqrt{\frac{4\pi^2 r}{a}}$$

$$= \sqrt{\frac{4\pi^2 (200)}{10}}$$

$$= 28 \text{ s}$$

On Earth, the ground pushes up on us. In the space station, the outer wall pushes the person in (centripetal force). This means that the person would feel as though the wall is the ground, and the direction opposite to the centripetal force is down.

43.



$$M_{\text{moon}} = 7.35 \times 10^{22} \text{ kg}$$

$$M_{\text{Earth}} = 5.98 \times 10^{24} \text{ kg}$$

$$r_{\text{from Earth}} = x$$

$$r_{\text{from moon}} = 3.3 \times 10^8 - x$$

$$F_{\text{moon}} = F_{\text{Earth}}$$

$$\frac{\not{G} M_{\text{moon}} \not{m}}{r_m^2} = \frac{\not{G} M_{\text{Earth}} \not{m}}{r_E^2}$$

$$\frac{7.35 \times 10^{22}}{(3.3 \times 10^8 - x)^2} = \frac{5.98 \times 10^{24}}{x^2}$$

$$(7.35 \times 19^{22})\, x^2 = (5.98 \times 10^{24})\,(3.38 \times 10^8 - x)^2$$

$$(7.35 \times 10^{22})\, x^2 = (5.98 \times 10^{24})\,((3.38 \times 10^8)^2 - 2(3.38 \times 10^8)x + x^2)$$

$$x^2 = 81.36(1.14 \times 10^{17} - (6.76 \times 10^8)\, x + x^2)$$

$$0 = 80.36x^2 - (5.5 \times 10^{10})\, x + 9.28 \times 10^{18}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{5.5 \times 10^{10} \pm \sqrt{(5.5 \times 10^{10})^2 - 4(80.36)(9.28 \times 10^{18})}}{2(80.36)}$$

$$= 3.83 \times 10^8,\ 3.02 \times 10^8$$

$x = 3.02 \times 10^8$ m is correct because the spacecraft must be between the Earth and the moon.

44. A geostationary satellite cannot remain above Melbourne as the net force on the satellite (the gravitational force) is towards the centre of the Earth, *not* towards the centre of the cirle mapped out by Melbourne as it rotates.