Unit 2

Area of study 1: Movement

Chapter 9

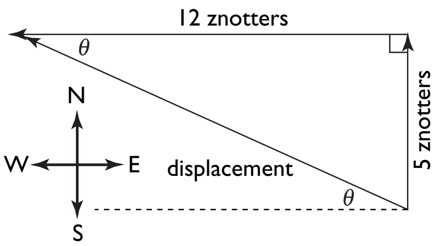
Analysing movement

1. (b) displacement, (d) velocity and (e) acceleration. In each case, both magnitude and direction are required for a complete description.

2. Only a finite time Δt can be measured with a stop-watch. Thus, only the average velocity $\frac{\Delta \boldsymbol{v}}{\Delta t}$ can be determined. Instantaneous velocity is the velocity at an instant of time. There is no time interval to measure with a stopwatch.

3. (a) distance = 5 znotters + 12 znotters = 17 znotters

(b)



magnitude of displacement $= \sqrt{5^2 + 12^2}$
$= 13$ znotters

$\tan \theta = \frac{5}{12}$

$\Rightarrow \theta = 23°$

Displacement is 13 znotters at an angle of 23° north of west (270° + 23° = 293° True).

(c) average velocity $= \frac{\text{displacement}}{\text{time}}$

$= \frac{13 \text{ znotters}}{6.5 \text{ znitters}}$ at 23° north of west

$= 2$ znotter znitter^{-1} at 23° north of west

average speed $= \frac{\text{displacement}}{\text{time}}$

$= \frac{17 \text{ znotters}}{6.5 \text{ znitters}}$

$= 2.6$ znotter znitter^{-1}

(d) $a = \frac{\Delta v}{\Delta t}$

Units are therefore

$\frac{\text{znotter znitter}^{-1}}{\text{znitter}} = \text{znotter znitter}^{-2}$

4. $100 \text{ km h}^{-1} = \frac{100 \text{ km}}{1 \text{ h}}$

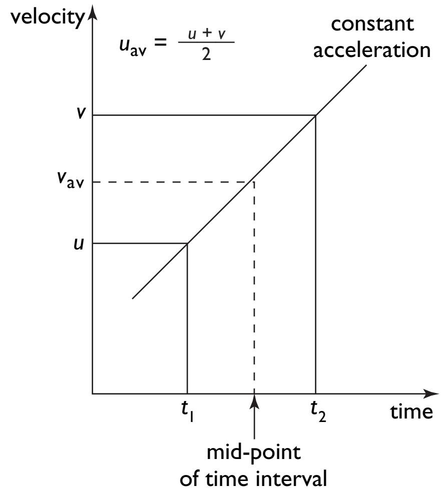
$= \frac{100\,000 \text{ m}}{3600 \text{ s}}$

$= 27.8 \text{ m s}^{-1}$

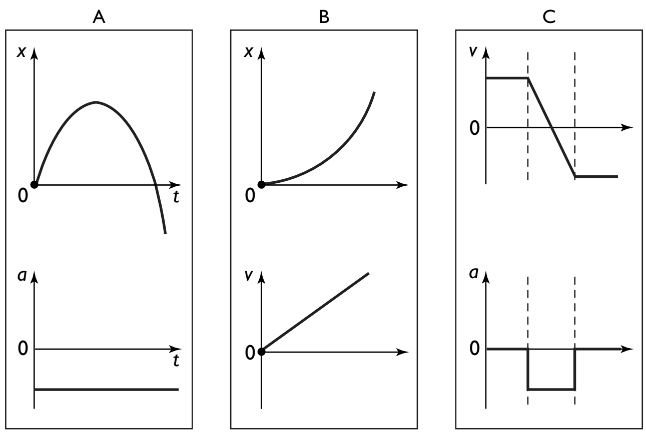
(or simply divide by 3.6)

5. (a) B, C
(b) B, D
(c) A, E
(d) A, E
(e) D

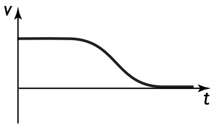
6. The instantaneous velocity is the same as the average velocity at the mid-point of the time interval during which the motion takes place.



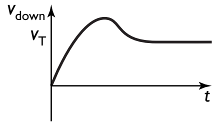
7. A: Constant negative acceleration with an initial positive velocity.
B: Constant positive acceleration from rest.
C: A constant positive velocity, followed by an interval of constant negative acceleration until a negative velocity equal in magnitude to the initial velocity is reached. The velocity then remains constant.



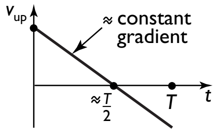
8. (a)



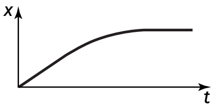
(b)



(c)



9. (a)



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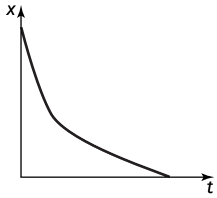
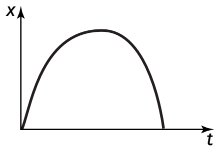


(c)



10. $1.5 \text{ m s}^{-1} = \dfrac{1.5 \text{ m}}{1 \text{ s}}$

$= \dfrac{0.0015 \text{ km}}{\frac{1}{3600} \text{ h}}$

$= 3600 \times 0.0015 \text{ km h}^{-1}$

$= 5.4 \text{ km h}^{-1}$

(or simply multiply by 3.6)

11. (a) $55 \text{ miles h}^{-1} = \dfrac{55 \text{ miles} \times 1.6 \text{ km/mile}}{1 \text{ h}}$

$= 88 \text{ km h}^{-1}$

(b) $88 \text{ km h}^{-1} = \dfrac{88\,000 \text{ m}}{3600 \text{ s}}$

$= 24 \text{ m s}^{-1}$

12. (a)

Event (m)	**Average speed (m s^{-1})**	**Calculations**
100	10.2	100 m/9.79 s
200	10.4	200 m/19.32 s
400	9.26	400 m/43.18 s
800	7.91	800 m/101.11 s (1 min 41.11 s = 101.11 s)
1500	7.28	1500 m/206.00 s (3 min 26.00 s = 206 s)
3000	6.81	3000 m/440.67 s (7 min 20.67 s = 440.67 s)
5000	6.58	5000 m/759.36 s (12 min 38.36 s = 759.36 s)
10 000	6.32	10 000 m/1582.75 s (26 min 22.75 s = 1582.75 s)

(b) The fact that the average speed during the 100 m event is similar to that during the 200 m event is due to the fact that the acceleration from rest to the maximum speed takes place over a significant fraction of the time taken for the 100 m event. Even though the maximum speed of the athlete is greater during the 100 m event, the average speed is not.

(c) $\text{average speed} = \dfrac{\text{distance travelled}}{\text{time interval}}$

$\Rightarrow \text{time interval} = \dfrac{\text{distance travelled}}{\text{average speed}}$

$= \dfrac{42\,200 \text{ m}}{7.28 \text{ m s}^{-1}}$

$= 5797 \text{ s}$

$= 96 \text{ min } 37 \text{ s}$

$= 1 \text{ h } 36 \text{ min } 37 \text{ s}$

(d) Only Maurice Green's. His event is the only one that involves only straight line motion.

13. (a) $\text{average speed} = \dfrac{\text{distance travelled}}{\text{time interval}}$

$= \dfrac{3000 \text{ m}}{216.081 \text{ s}}$

$= 13.88 \text{ m s}^{-1}$

(b) $\text{time interval} = \dfrac{\text{distance travelled}}{\text{average speed}}$

$= \dfrac{151\,000 \text{ m}}{13.88 \text{ m s}^{-1}}$

$= 10\,879 \text{ s}$

$= 3.02 \text{ h}$

(c) $\text{time interval} = \dfrac{\text{distance travelled}}{\text{average speed}}$

$= \dfrac{151 \text{ km}}{80 \text{ km h}^{-1}}$

$= 1.9 \text{ h}$

(d) (i) $\text{average speed} = \dfrac{\text{distance travelled}}{\text{time interval}}$

$= \dfrac{302 \text{ km}}{4.0 \text{ h}}$

$= 76 \text{ km h}^{-1}$

(ii) $v_{av} = \dfrac{\Delta x}{\Delta t}$

$= 0 \text{ km h}^{-1}$

since Δx (displacement) = 0

14. $\text{Time for tortoise} = \dfrac{\text{distance travelled}}{\text{average speed}}$

$= \dfrac{1000 \text{ m}}{0.075 \text{ m s}^{-1}}$

$= 13\,333 \text{ s}$

Time for hare at maximum speed

$= \dfrac{\text{distance travelled}}{\text{average speed}}$

$= \dfrac{1000 \text{ m}}{20 \text{ m s}^{-1}}$

$= 50 \text{ s}$

In a tied race, the hare must have napped for

$13\,333 \text{ s} - 50 \text{ s} = 13\,283 \text{ s}$

$= 3.7 \text{ h}$ (approx. 3 h 41 min)

15. average speed $= \dfrac{\text{distance travelled}}{\text{time interval}}$

(not $\dfrac{v_1 + v_2 + v_3}{3}$!)

$= \dfrac{120 + 120 + 120 \text{ m}}{20 + 30 + 60 \text{ s}}$

$= \dfrac{360 \text{ m}}{110 \text{ s}}$

$= 3.3 \text{ m s}^{-1}$

16. (a) Predictions will vary but likely response is 90 km h^{-1} — incorrectly obtained by

$\dfrac{80 \text{ km h}^{-1} + 100 \text{ km h}^{-1}}{2}$

(b) average speed $= \dfrac{\text{distance travelled}}{\text{time interval}}$

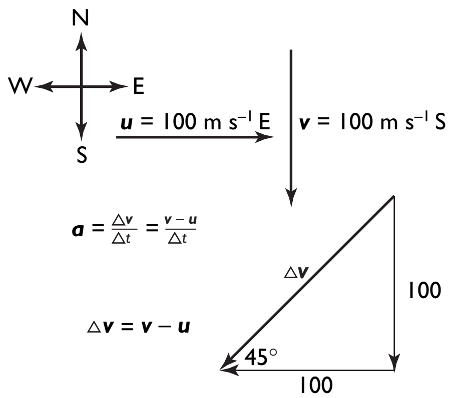
$= \dfrac{300 + 300}{\frac{300}{80} + \frac{300}{100}}$

$= 89 \text{ km h}^{-1}$

17.

	(i) change in speed	(ii) change in velocity
(a)	−40 km h^{-1}	40 km h^{-1} south (or −40 km h^{-1} north)
(b)	−20 m s^{-1}	−20 m s^{-1} in original direction (or +20 m s^{-1} opposite to original direction)
(c)	+5 m s^{-1}	−55 m s^{-1} in original direction (or +55 m s^{-1} opposite to original direction)

18. Yes, there is an acceleration. Even though the speed has not changed, the velocity has changed.



The magnitude of $\Delta \boldsymbol{v}$ = 141 m s^{-1}. Its direction is south-west. The acceleration is therefore not zero.

19. (a) B

(b) A, D, E (the intervals in which the gradient is positive)

(c) 40 s (the first instant at which the skateboard rider moves in a southerly direction)

(d) 20 m north (change in position after 80 s = 20 m north of starting point)

(e) 260 m (80 m in northerly direction, followed by 120 m in a southerly direction, followed by 60 m in a northerly direction)

(f) D (the gradient is increasing)

(g) E (the gradient is decreasing)

(h) average speed $= \dfrac{\text{distance travelled}}{\text{time interval}}$

$= \dfrac{260 \text{ m}}{80 \text{ s}}$

$= 3.3 \text{ m s}^{-1}$

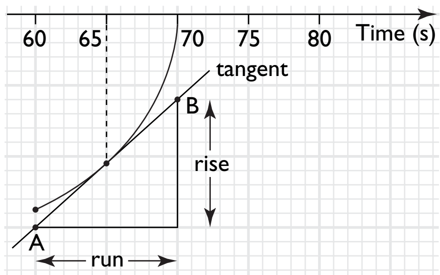
(i) $\boldsymbol{v}$ = gradient

$= \dfrac{-120 \text{ m}}{20 \text{ s}}$

$= -6.0 \text{ m s}^{-1}$

$= 6.0 \text{ m s}^{-1}$ south

(j) $\boldsymbol{v}$ = gradient at time t = 65 s



To find the gradient a tangent must be drawn on the curve at t = 65 s. Two convenient points need to be drawn on the tangent (e.g. A and B).

gradient $= \dfrac{\text{rise}}{\text{run}}$

$= \dfrac{-18 - (-48)}{70 - 60}$ (approx.)

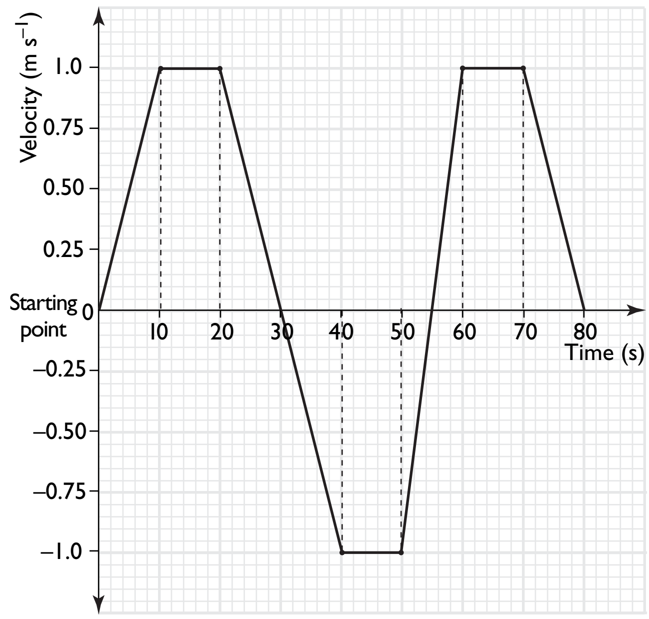
$= 3 \text{ m s}^{-1}$

The direction is north.

20. (a) B, D, F (gradient = 0 during these sections. That is, there is no change in velocity.)

(b) Displacement = total area under graph.

This is most easily calculated by dividing the graphs into triangles and rectangles as shown in the figure below.



Total area

$= \frac{1}{2} \times 10 \times 1 + 10 \times 1 + \frac{1}{2} \times 10 \times 1 - \frac{1}{2} \times 10 \times 1 - 10 \times 1 - \frac{1}{2} \times 5 \times 1 + \frac{1}{2} \times 5 \times 1 + 10 \times 1 + \frac{1}{2} \times 10 \times 1$

$= 5 + 10 + 5 - 5 - 10 - 2.5 + 2.5 + 10 + 5$

$= +20$ m

(c) $v_{av} = \dfrac{\Delta \boldsymbol{x}}{\Delta t}$

$= \dfrac{+20 \text{ m}}{80 \text{ s}}$

$= 0.25 \text{ m s}^{-1}$

(d) 30 s (the instant that the velocity becomes negative)

(e) It didn't. (The negative displacement that occurs between 30 s and 55 s is not as great as the positive displacement between 0 s and 30 s.)

(f) C, G (when the gradient is negative)

(g) The first half of interval C, the first half of interval E and interval G. (During these periods, the magnitude of the velocity is decreasing.)

(h) A negative acceleration doesn't always decrease the speed and a positive acceleration doesn't always increase the speed. A negative acceleration increases the speed if the velocity is negative and decreases the speed if the velocity is positive. Similarly, a positive acceleration decreases the speed if the velocity is negative and increases the speed if the velocity is positive.

(i) a = gradient

$= \dfrac{+2.0 \text{ m s}^{-1}}{10 \text{ s}}$

$= 0.20 \text{ m s}^{-2}$

(j) $a_{av} = \dfrac{\Delta v}{\Delta t}$

$= \dfrac{1.0 \text{ m s}^{-1}}{20 \text{ s}}$

$= 0.050 \text{ m s}^{-2}$

(k) The motion of the toy robot can be described in nine different intervals.

First 10 seconds: The toy robot started from rest and increased its speed at a constant rate until reaching a speed of 1.0 m s^{-1} after 10 seconds.

10 s–20 s: It maintained a constant speed of 1.0 m s^{-1}.

20 s–30 s: It slowed down at a constant rate. It was at rest for an instant, 30 seconds after starting.

30 s–40 s: It increased its speed at the same constant rate as the first interval, but in the opposite direction to reach a maximum speed of 1.0 m s^{-1}.

40 s–50 s: It maintained a constant speed of 1.0 m s^{-1}.

50 s–55 s: It decelerated to rest at a constant rate.

55 s–60 s: It increased its speed at a constant rate in the original direction. The acceleration was twice that of the first interval.

60 s–70 s: It maintained a constant speed of 1.0 m s^{-1}.

70 s–80 s: It decelerated to rest at a constant rate.

21. (a) 3.0 s (can be read directly from the graph)

(b) $\boldsymbol{a}$ = gradient

$= \dfrac{10 \text{ m s}^{-1}}{4 \text{ s}}$

$= 2.5 \text{ m s}^{-2}$

(c) Let T = time at which stuntman catches the bus. At time T the displacement of the stuntman is equal to the displacement of the bus.

⇒ area under stuntman graph = area under bus graph

$\Rightarrow \frac{1}{2} \times 4 \times 10 + 10(T - 4) = 8T$

$\Rightarrow 20 + 10T - 40 = 8T$

$\Rightarrow 2T = 20$

$\Rightarrow T = 10 \text{ s}$

(d) Distance = magnitude of displacement

= area under graph

= $8T$ (or $10T - 20$)

= 80 m

22. (a) A constant speed is reached when the acceleration becomes zero. The acceleration of the jet ski becomes zero first, after 8.0 s.

(b) (i) Δv = area under acceleration vs time graph

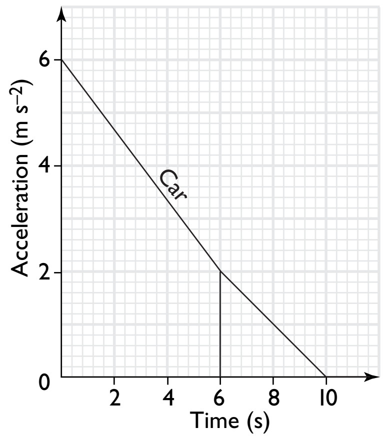
$= \frac{1}{2} \times 8 \times 4$

$= 16 \text{ m s}^{-1}$

$v = 5.0 \text{ m s}^{-1}$

$v = 21 \text{ m s}^{-1}$ for jet ski

(ii) Dividing the graph into a trapezium and triangle as shown in the figure below.



$\Delta \boldsymbol{v}$ = area under acceleration vs time graph

$= \dfrac{6 + 2}{2} \times 6 + \frac{1}{2} \times (10 - 6) \times 2$

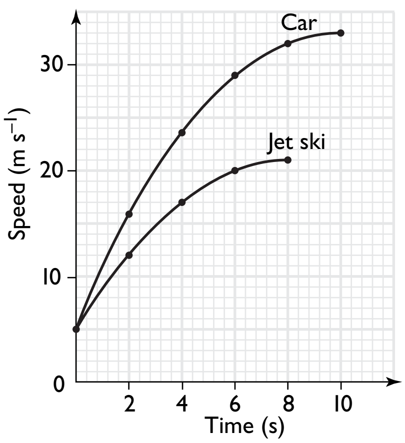
$= 24 + 4$

$= 28 \text{ m s}^{-1}$

$v = 5.0 \text{ m s}^{-1}$

$\Rightarrow v = 33 \text{ m s}^{-1}$ for car

(c)



Speed vs time graph for jet ski and car

23. For a car that accelerates from rest to 60 km h^{-1} (17 m s^{-1}) in say 5 seconds:

$$a = \frac{\Delta \boldsymbol{v}}{\Delta t} = \frac{17 \text{ m s}^{-1}}{5 \text{ s}}$$
$$= 3 \text{ m s}^{-2}$$

24. v_{av} for a 100 m sprint in 9.84 m s^{-1} = 10.2 m s^{-1}
estimate v_{max} = 12 m s^{-1} is reached after 2 seconds.

$$a = \frac{\Delta \boldsymbol{v}}{\Delta t} = \frac{12 \text{ m s}^{-1}}{2 \text{ s}}$$
$$= 6 \text{ m s}^{-2}$$

25. (a) $a = 6.0$ m s^{-2}
$u = 17$ m s^{-1}
$v = 28$ m s^{-1} $t = ?$
$v = u + at$
$\Rightarrow 28 = 17 + 6.0t$
$$\Rightarrow t = \frac{28 - 17}{6.0}$$
$= 1.8$ s

(b) $a = 2.0$ m s^{-2}
$u = 0$
$v = 10$ m s^{-1} $t = ?$
$v = u + at$
$\Rightarrow 10 = 0 + 2.0t$
$$\Rightarrow t = \frac{10}{2.0}$$
$= 5.0$ s

26. $a = 10$ m s^{-2}
$x = 36$ m
$u = 0$
(a) $t = ?$
$$x = ut + \tfrac{1}{2}at^2$$
$\Rightarrow 36 = 0 + 5t^2$
$$\Rightarrow t = \sqrt{\frac{36}{5}}$$
$= 2.7$ s

(b) $v = ?$
$v^2 = u^2 + 2ax$
$= 0 + 2 \times 10 \times 36$
$= 720$
$\Rightarrow v = \sqrt{720}$
$= 27$ m s^{-1}

27. $x = 12$ m
$t = 2$ s
$v = 0$
(a) $u = ?$
$$x = \frac{u + v}{2}t$$
$$\Rightarrow 12 = \frac{u}{2} \times 2$$
$\Rightarrow u = 12$ m s^{-1}

(b) $a = ?$
$$x = vt - \tfrac{1}{2}at^2$$
$\Rightarrow 12 = 0 - \tfrac{1}{2} \times a \times 4$
$\Rightarrow 2a = -12$
$a = -6.0$ m s^{-2}

28. $u = 100$ km h^{-1} = 27.7 m s^{-1}
$v = 0$
(a) $x = 48$ m
$a = ?$
$v^2 = u^2 + 2ax$
$\Rightarrow 0 = (27.7)^2 + 2 \times a \times 48$
$$\Rightarrow a = -\frac{(27.7)^2}{96}$$
$= -8.0$ m s^{-2}

(b) $x = 48$ m
$t = ?$
$$x = \frac{u + v}{2}t$$
$$\Rightarrow 48 = \frac{27.7}{2}t$$
$$\Rightarrow t = \frac{96}{27.7}$$
$= 3.5$ s

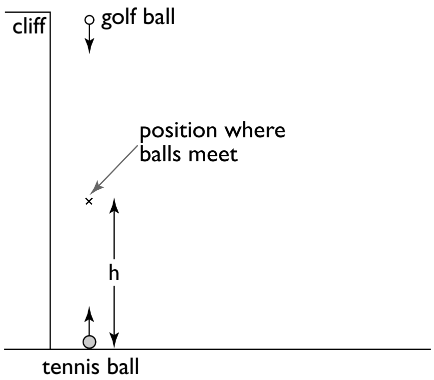
(c) The reaction time of the driver needs to be known so that the distance travelled between the instant that the branch is seen and the instant that the brakes are applied can be determined. An estimate of 0.2 s would be reasonable for the reaction time. At a constant speed of 100 km h^{-1} (27.7 m s^{-1}) the car would travel a distance of 27.7 m s^{-1} × 0.2 s = 5.5 m. The total distance required to stop is therefore 5.5 m (reacting distance) + 48 m (braking distance) = 53.5 m. The car would not stop in time.

29. In order to make the leap, the dancer must rise and fall in 0.5 s.
Ignoring air resistance, the fall takes the same amount of time as the rise, –0.25 s.
The time taken for the dancer to rise to a height of 80 cm can be calculated:
$v = 0$, $x = 0.80$ m, $a = -10$ m s^{-2}
$t = ?$
$$x = vt - \tfrac{1}{2}at^2$$
$\Rightarrow 0.80 = 5t^2$
$$\Rightarrow t = \sqrt{\frac{0.80}{5}}$$
$= 0.4$ s
The time taken for the dancer to fall from a height of 80 cm can be calculated.
$u = 0$, $x = 0.80$ m, $a = 10$ m s^{-2}
$t = ?$
$$x = ut + \tfrac{1}{2}at^2$$
$0.80 = 5t^2$
$$\Rightarrow t = \sqrt{\frac{0.80}{5}}$$
$= 0.4$ s
That is, without knowing the 'take off' speed of the dancer it can be shown that the leap would take 0.8 s. The leap is not possible.

30. It is important to remember that at the instant that the Rolls Royce rolls off the truck it is moving in the same direction as the truck.

After one minute it has moved a distance of 1000 m (a constant speed of 60 km h^{-1} is equal to 1 km min^{-1}). During the driver's reaction time the distance moved by the truck is 8.3 m (the distance moved in 0.5 s at a constant speed of 60 km h^{-1} = 16.67 m s^{-1} × 0.5 s).
The braking distance of the truck is 25 m.
The total distance moved by the truck is 1000 m + 8.3 m + 25 m = 1033 m.
The distance moved by the Rolls Royce is 240 m (in the same direction as that of the truck). The Rolls Royce is therefore 793 m behind the stopped truck.

31.



(a) The balls collide when the tennis ball is at the top of its path at a time t_c.
For the tennis ball: $v = 0$

$a = -10$ m s^{-2} (taking up as positive)
$x = h$
$t = t_c$
$u = ?$

$$x = vt + \frac{1}{2}at^2$$

$\Rightarrow h = 5t^2$ (1) (since $a = -10$ m s^{-2})

For the golf ball: $u = 0$

$a = 10$ m s^{-2} (taking down as positive)
$x = 100 - h$
$t = t_c$

$$x = ut + \frac{1}{2}at^2$$

$\Rightarrow 100 - h = 5t^2$ (2)

Add (1) and (2).

$$\Rightarrow 100 = 10t^2$$
$$\Rightarrow t = \sqrt{10}$$
$$= 3.15 \text{ s}$$

For the tennis ball:

$$v = u + at$$
$$\Rightarrow 0 = u - 10 \times 3.16$$
$$\Rightarrow u = 32 \text{ m s}^{-1}$$

(b) Substitute in (1)

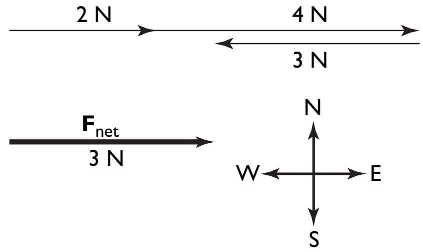
$$\Rightarrow h = 5(3.16)^2$$
$$= 50 \text{ m}$$

The balls meet 50 m from the ground.

Chapter 10

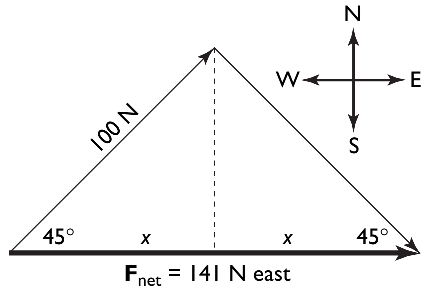
Forces in action

1. 'My mass is 75 kg', 'My weight is 750 N'.
2. Vector quantities have magnitude, unit and direction. Scalar quantities have magnitude and unit.
3. (b) weight, and (c) gravitational field strength.
4. Weight is a vector quantity. On the surface of the Earth its magnitude would be constant (to 2 sig. fig.) but the direction changes from place to place. There is not enough information given to compare its direction on Earth with its direction on Mars.
5. (a)



The net force is 3 N to the east.

(b)



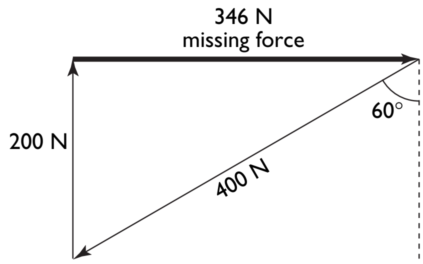
The answer can be obtained by scale vector diagram or using trigonometry.

$$\text{i.e. } \cos 45° = \frac{x}{100 \text{ N}}$$
$$\Rightarrow x = 100 \text{ N} \cos 45°$$
$$= 70.7 \text{ N}$$

Magnitude of $\boldsymbol{F}_{net} = 2x$

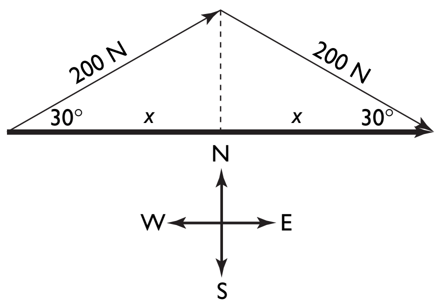
$$\boldsymbol{F}_{net} = 1.4 \times 10^2 \text{ N east}$$

6. (a)



Answer is best obtained by scale vector diagram.
Missing force = 346 N east

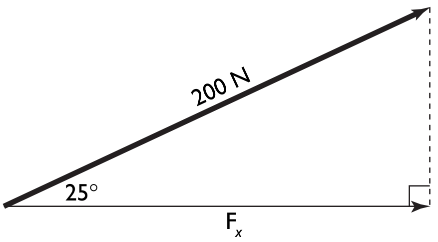
(b) The sum of the two forces acting at 30° to the horizontal can be obtained by scale vector diagram or using trigonometry.



$$\cos 30° = \frac{x}{200 \text{ N}}$$
$$\Rightarrow x = 200 \text{ N} \cos 30°$$
$$= 173.2 \text{ N}$$
$$\Rightarrow \text{sum} = 346.4 \text{ N east}$$

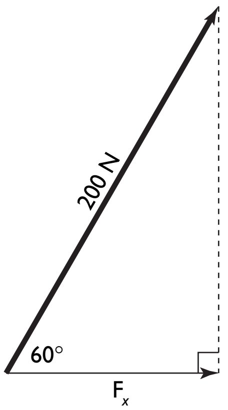
Adding this to the 200 N force to the west gives a total force of 146.4 N east.
In order to obtain a net force of 200 N east, an additional force of 53.6 N east is required.

7. (a)



$$\cos 25° = \frac{F_x}{200\text{ N}}$$
$$\Rightarrow F_x = 200\text{ N} \cos 25°$$
$$= 1.8 \times 10^2\text{ N}$$

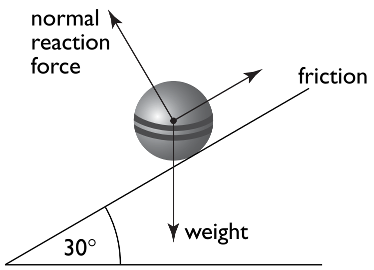
(b)



$$\cos 60° = \frac{F_x}{200\text{ N}}$$
$$\Rightarrow F_x = 200\text{ N} \cos 60°$$
$$= 100\text{ N}$$

(c) 0 N

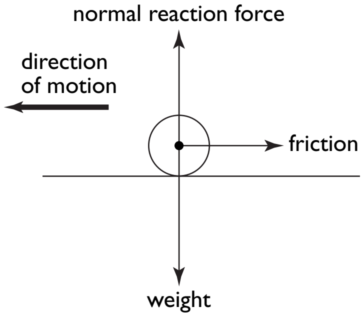
8. (a)



(b) The direction of the net force on the ball is down the hill (parallel to the hill) since the ball is speeding up as it rolls down the hill.

(c) Weight force; if it were not, the net force could not be down the hill.

(d) The ball slows to a stop because of the effect of the friction acting on the ball. On a horizontal surface, the normal reaction is equal in magnitude to the weight.



9. It describes what happens to an object when acted on by a net force equal to zero. The word *inertia* is used to describe the tendency of an object to resist change.

10. The force of the Earth pushes on the tyres in the opposite direction to the force applied by you pushing the car..

11. (a) 0 m s^{-1}

(b) 10 m s^{-2}. This is the same as the acceleration throughout the rest of its flight.

(c) 5.0 N down. The net force on the ball throughout its flight (assuming air resistance is negligible) = $m\boldsymbol{g}$ = 0.50 kg × 10 m s^{-2} down.

12. (*from top to bottom*)
The wall pushes on your palm in the opposite direction.
The bicycle pedal pushes up on your foot.
You push down on the ground when you are standing.
Your body pulls up on the Earth.
The broken-down car pushes on you in the opposite direction.
The nail pushes up on the hammer.

13. Idealisations can be made to allow the use of a simple mathematical model to solve a physical problem. For example, in order to use simple equations to analyse the motion of a falling ball, the idealisation can be made that the air resistance is insignificant and the ball does not spin.

14. (a) 40 km h^{-1} = $\frac{40}{3.6}$ m s^{-1}

$$\boldsymbol{a} = \frac{\Delta \boldsymbol{v}}{\Delta t}$$
$$= \frac{11.11 - 0}{3.2}$$
$$= 3.47\text{ m s}^{-2}$$
$$\boldsymbol{F}_{net} = m\boldsymbol{a}$$
$$= 1000\text{ kg (estimate)} \times 3.47\text{ m s}^{-2}$$
$$= 3 \times 10^3\text{ N}$$

(b) At terminal velocity
air resistance = weight
$= m\boldsymbol{g}$
$= 80\text{ kg} \times 10\text{ N kg}^{-1}$
$= 8 \times 10^2\text{ N}$

(c) $p = m\boldsymbol{v}$
$= 80$ kg (estimate) × 10 m s^{-1} (estimate)
$= 8 \times 10^2$ kg m s^{-1}

(d) $p = m\boldsymbol{v}$
$= 800$ kg × 60 km h^{-1}
$= 800$ kg × 16.7 m s^{-1}
$= 1 \times 10^4$ kg m s^{-1}

(e) Impulse = $m\Delta\boldsymbol{v}$
$= 70$ kg × 8 m s^{-1}
$= 6 \times 10^2$ kg m s^{-1} or 6×10^2 N s

(f) Impulse = $m\Delta\boldsymbol{v}$
$= 0.4$ kg (estimate) × 5 m s^{-1}
$= 2$ N s

(g) $\Delta p = m\Delta\boldsymbol{v}$
$= 0.2$ kg (estimate) × 50 m s^{-1} (estimate)
$= 10$ kg m s^{-1}

15. (a) $F_{net} = ma$
$= 2.2 \times 10^6$ kg × 3.0 m s^{-2}
$= 6.6 \times 10^6$ N

(b) F_{net} = thrust − weight
$\Rightarrow 6.6 \times 10^6$ N = thrust − 2.2×10^7 N
$\Rightarrow$ Thrust = 2.9×10^7 N

16. (a) Both experience the same acceleration. The acceleration is given by $\boldsymbol{a} = \frac{\boldsymbol{F}_{net}}{m}$. The air resistance on these objects is insignificant when compared to their weight and can be ignored. Therefore, both the net force (weight) and mass of the gold bar are 10 times as great as they are for the bowling ball.

(b) The air resistance in the doormat is significant when compared with its weight. Therefore, the net force on the doormat is less than that of the bowling ball (which has the same mass as the doormat) and the acceleration of the doormat $\frac{\boldsymbol{F}_{net}}{m}$ is smaller than that of the bowling ball (and the gold bar).

17. Taking down as positive:

(a) $\Delta p = m\Delta \boldsymbol{v}$
$= 0.060(-6.0 - 8.0)$
$= -0.84$ kg m s^{-1}
$= 0.84$ kg m s^{-1} up

(b) 0.84 kg m s^{-1} or 0.84 N s down
The impulse applied by the tennis ball to the ground is equal and opposite to the impulse applied by the ground to the tennis ball.

(c) No. The ground does not move a measurable amount. The impulse applied to the ground is 0.84 kg m s^{-1} but the mass of the Earth is so large that the change in velocity is negligible.

(d) $F_{net}\Delta t = \Delta p$
$$\Rightarrow F_{net} = \frac{\Delta p}{\Delta t} = \frac{0.84 \text{ N s up}}{2.0 \times 10^{-3} \text{ s}} = 4.2 \times 10^2 \text{ N up}$$

(e) $F_{net} = N - mg$
$\Rightarrow 4.2 \times 10^2 = N - 0.06 \times 10$
$= N - 0.6$
$\Rightarrow N = 4.2 \times 10^2 + 0.6$
$= 4.2 \times 10^2$ N up

18. (a) $W = mg$
$= 1400$ kg $\times$ 10 N kg^{-1}
$= 1.4 \times 10^4$ N

(b) On Mars: $W = mg$
$= 1400$ kg $\times$ 3.6 N kg^{-1}
$= 5.0 \times 10^3$ N

(c) m = 1400 kg anywhere. It is a measure of the amount of matter in an object or substance and does not depend on the gravitational field strength.

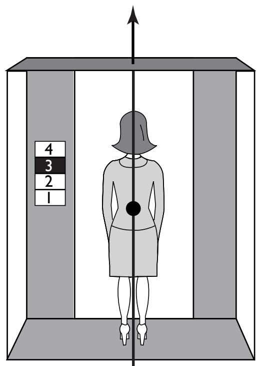
19. (a) apple: $m \simeq 0.1$ kg $\Rightarrow W \simeq 0.1$ kg $\times$ 10 N kg^{-1}
$\simeq 1.0$ N

(b) this book: $m \simeq 1$ kg $\Rightarrow W \simeq 1$ kg $\times$ 10 N kg^{-1}
$\simeq 10$ N

(c) if physics teacher has mass of about 80 kg:
$W \simeq 80$ kg $\times$ 10 N kg^{-1}
$\simeq 800$ N

20. Assume mass of student is m kg.
(a) $10m$ N
(b) $3.6m$ N
(c) m kg
If, for example, $m = 65$ kg:
(a) At Earth's surface $W = 65$ kg $\times$ 10 N kg^{-1} = 650 N
(b) On the surface of Mars $W = 65$ kg $\times$ 3.6 N kg^{-1}
= 234 N
(c) $m = 65$ kg on Mars and anywhere else. It does not depend on the gravitational field strength.

21.



The forces acting on you in an elevator

The net force must be in the direction in which the elevator is speeding up.

(a) (i) Normal reaction force is equal to your weight and the net force is zero.
(ii) Normal reaction force is equal to your weight. The speed is not changing so the net force must be zero.
(iii) Normal reaction force is greater than your weight because the speed of the elevator is increasing in an upwards direction.
(iv) Normal reaction force is less than your weight because the net force must be down to make the elevator decrease its upward speed.

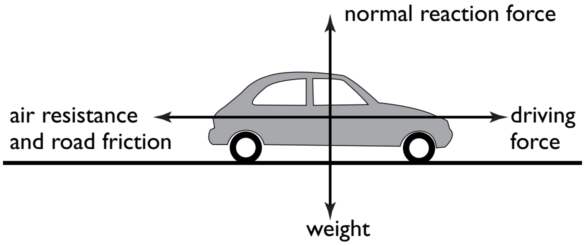
(b) You feel the sensation of weight only if there is an upward push on you by an object like the ground, a floor, a chair or a bed. The apparent weight that you feel is the size of this upward push. If the elevator is accelerating upwards, the normal reaction force is greater than your weight and you feel heavier. If the elevator is accelerating downwards, the normal reaction force must be less than your weight and you feel lighter.

22. (a) 70 kg $\times$ 10 N kg^{-1} = 700 N
(b) (i) The upwards force is greater when the jumper is decelerating downwards or, in other words, accelerating upwards.

(ii) The weight is greater than the upwards pull when the jumper is accelerating downwards.

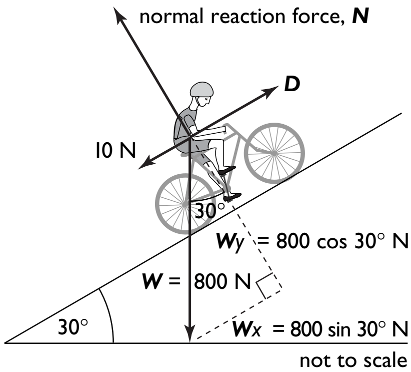
(c) The tension in the bungee cord must be equal in magnitude to the jumper's weight in order for the speed to be constant, that is, 700 N. This occurs only for an instant during the fall. At this instant the cord is extending and the tension is increasing.

23. (a)



(b) The car is in uniform motion. Therefore the net force must be zero.

24.



(a) The net force is zero because the motion is uniform.

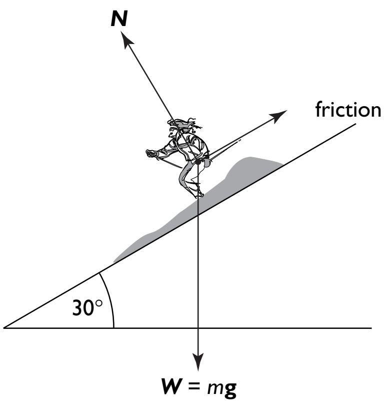
(b) $W = mg = 80\text{ kg} \times 10\text{ N kg}^{-1}$
$= 800\text{ N}$
$W_x = W \sin 30°$
$= 800 \sin 30°$
$= 400\text{ N}$

(c) $\boldsymbol{F}_{net} = 0$
Therefore, the sum of forces parallel to the slope is zero.
$\Rightarrow D - 400 - 10 = 0$
$\Rightarrow D = 410\text{ N}$

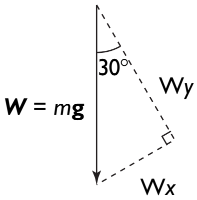
(d) The sum of the forces perpendicular to the slope is zero.
$\Rightarrow N = W_y$
$= 800 \cos 30°$
$= 6.9 \times 10^2\text{ N}$

25. (a) Since the speed is increasing down the slope, the net force must be in the same direction, that is, down the slope.

(b)



(c) $W_x = mg \sin 30°$
$= 60 \times 10 \times \sin 30°$
$= 300\text{ N}$



(d) $\boldsymbol{F}_{net} = 300\text{ N down slope} + 8\text{ N up slope}$
$= 292\text{ N down slope}$
Magnitude of net force = 292 N

26. The vehicle experiences a non-zero net force that slows it down. No such force acts on you. The net force on you is zero. Therefore, you continue in your state of constant velocity. You are not really thrown forward. You continue your motion while the vehicle slows down.

27. There is an unbalanced force on the bike and its velocity changes. Your inertia keeps you moving forward as there is no unbalanced force to change your motion (apart from gravity).

28. (a) $F_{net} = 10\,000\text{ N} - 2500\text{ N}$
$= 7500\text{ N}$

(b) $a = \dfrac{F_{net}}{m}$
$= \dfrac{7500}{1200}$
$= 6.3\text{ m s}^{-2} \quad (6.25\text{ m s}^{-2})$

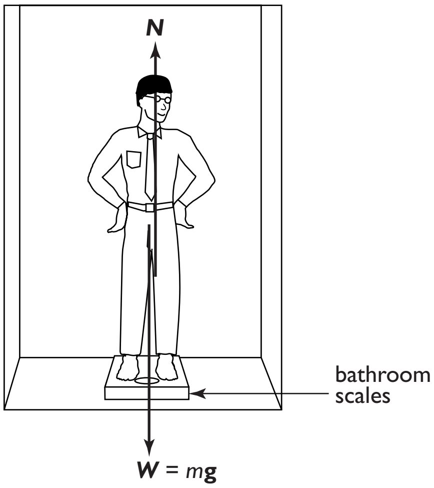
(c) $u = 0,\ a = 6.25\text{ m s}^{-2},\ t = 5.0\text{ s}$
$v = u + at$
$= 0 + 6.25 \times 5.0$
$= 31\text{ m s}^{-1}$

(d) $x = ut + \frac{1}{2}at^2$
$= 0 + \frac{1}{2} \times 6.25 \times (5.0)^2$
$= 78\text{ m}$

29. $u = 25\text{ m s}^{-1},\ v = 0,\ x = 360\text{ m}$
$\boldsymbol{v}^2 = u^2 + 2ax$
$\Rightarrow 0 = (25)^2 + 720a$
$\Rightarrow a = -\dfrac{625}{720}$
$= -0.868\text{ m s}^{-2}$
Frictional force = net force
$= ma$
$= 8.0 \times 10^6 \times -0.868$
$= 6.9 \times 10^6\text{ N}$

30. $x = 50\text{ m},\ u = 12\text{ m s}^{-1},\ v = 0\text{ m s}^{-1}$
$v^2 = u^2 + 2ax$
$\Rightarrow 0 = (12)^2 + 100a$
$\Rightarrow a = -\dfrac{144}{100}$
$= -1.44\text{ m s}^{-2}$
Frictional force = net force
$= ma$
$= 70 \times 1.44$
$= 1.0 \times 10^2\text{ N} \quad (101\text{ N})$

31.



The forces acting on the teacher

The force N is provided by the bathroom scales. The reading on the scales will be equal to N.

When the teacher is stationary N = mg = 700 N

(a) $\boldsymbol{F}_{net} = 0$ since velocity is constant

$\Rightarrow N - mg = 0$

$\Rightarrow N = mg$

$= 700$ N

(b) Assign down as negative.

$\boldsymbol{a} = -2.0 \text{ m s}^{-2}$

$N - mg = ma$

$mg = 700$

$\therefore\ m = 70$ kg (since $g = 10 \text{ N kg}^{-1}$)

$N - 700 = 70 \times -2.0$

$\Rightarrow N = -140 + 700$

$= 560$ N

(c) $\boldsymbol{a} = 2.0 \text{ m s}^{-2}$

$N - mg = ma$

$\Rightarrow N - 700 = 70 \times 2.0$

$\Rightarrow N = 140 + 700$

$= 840$ N

32. Assigning up as positive, the net force on the lift is given by

$\boldsymbol{F}_{net} = T - W$

where T is the tension in the cable and

W is the weight of the lift and its passengers.

$W = mg$

$= (480 + 24 \times 70) \times 10$

$= 2160 \times 10$

$= 21\,600$ N

$\Rightarrow F_{net} = T - 21\,600$

$\Rightarrow ma = T - 21\,600$

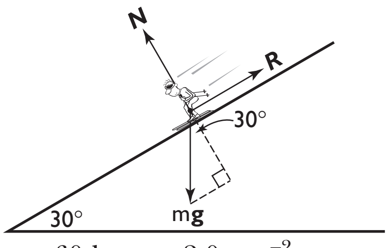
$\Rightarrow a = T - \dfrac{21\,600}{m}$

$T_{max} = 25\,000$ N

$\Rightarrow a_{max} = \dfrac{25\,000 - 21\,600}{2160}$

$= 1.6 \text{ m s}^{-2}$

33.



$m = 60$ kg, $\boldsymbol{a} = 2.0 \text{ m s}^{-2}$

The weight mg can be resolved into components that are parallel to and perpendicular to the slope.

Considering the forces parallel to the slope

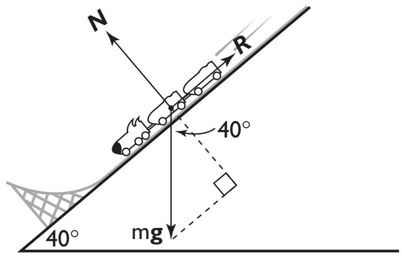
$F_{net} = mg \sin 30° - R$

$\Rightarrow ma = mg \sin 30° - R$

$\Rightarrow 120 = 600 \sin 30° - R$

$\Rightarrow R = 180$ N

34.



$m = 400$ kg, $R = 180$ N

The weight mg can be resolved into components that are parallel to and perpendicular to the slope.

Considering the forces parallel to the slope

$F_{net} = mg \sin 40° - 180$

$\Rightarrow 400\,a = 4000 \sin 40° - 180$

$\Rightarrow a = \dfrac{4000 \sin 40° - 180}{400}$

$= 6.0 \text{ m s}^{-2}$ (5.97 m s^{-2})

35. (a) $a = \dfrac{\Delta v}{\Delta t} = \dfrac{6}{8} = 0.75 \text{ m s}^{-2}$

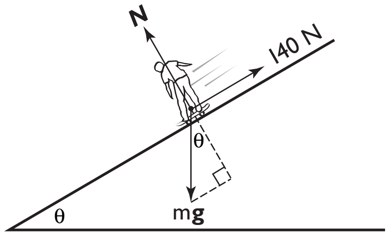
$m = 56$ kg

$F_{net} = ma$

$= 56 \text{ kg} \times 0.75 \text{ m s}^{-2}$

$= 42$ N

(b)



The weight, mg, can be resolved into components that are parallel to and perpendicular to the slope.

Considering the forces parallel to the slope:

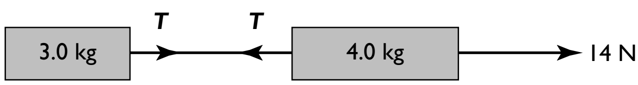
$\boldsymbol{F}_{net} = mg \sin\theta - 140$

$\Rightarrow 42 = 560 \sin\theta - 140$

$\Rightarrow \sin\theta = \dfrac{42 + 140}{560}$

$\Rightarrow \theta = 19°$

36.



(a) $F_{net} = 14$ N, $m = 7.0$ kg

$a = \dfrac{F_{net}}{m}$

$= \dfrac{14 \text{ N}}{7.0 \text{ kg}}$

$= 2.0 \text{ m s}^{-2}$

(b) Consider the 3.0 kg trolley:

$F_{net} = T$

$\Rightarrow 3.0 \text{ kg} \times 2.0 \text{ m s}^{-2} = T$

$\Rightarrow T = 6.0$ N

(c) $F_{net} = 14 \text{ N} - T$

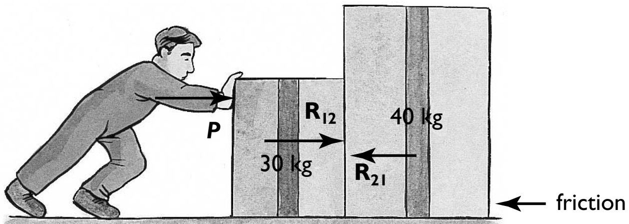
$= 14 \text{ N} - 6.0 \text{ N}$

$= 8.0$ N

(d) $F_{net} = 14$ N
$m = 4.0$ kg
$$a = \frac{F_{net}}{m}$$
$$= \frac{14 \text{ N}}{4.0 \text{ kg}}$$
$= 3.5$ m s^{-2}

37. (a) traction (friction) on your blades
(b) the component of gravity down the slope and the force of the ground on your poles if you use them
(c) the tension in the rope attached to the handle that you are desperately holding
(d) the traction (friction) as you push back on the ground
(e) the force exerted by the water on your hands, arms, legs and feet as you push back with your hands and kick
(f) the force exerted by the water on the oar as the oar pushes back on the water

38.



$\mathbf{R_{12}}$ = force exerted by 30 kg crate on 40 kg crate
$\mathbf{R_{21}}$ = force exerted by 40 kg crate on 30 kg crate

Assign direction to the right as positive.

(a) Applying Newton's Second Law to the system of the two crates:
$$\boldsymbol{F}_{net} = m\boldsymbol{a}$$
$$\Rightarrow P - \text{friction} = 70\boldsymbol{a}$$
$$\Rightarrow 420 - (70 \times 2.0) = 70\boldsymbol{a}$$
$$\Rightarrow \boldsymbol{a} = \frac{420 - 140}{70}$$
$= 4.0$ m s^{-2} to the right

(b) Applying Newton's Second Law to the 40 kg crate:
$\boldsymbol{F}_{net} = m\boldsymbol{a}$
$= 40 \times 4.0$
$= 160$ N to the right

(c) Applying Newton's Second Law to the 30 kg crate:
$\boldsymbol{F}_{net} = m\boldsymbol{a}$
$= 30 \times 4.0$
$= 120$ N
$\boldsymbol{F}_{net} = P + R_{21}$ + friction on 30 kg crate
$\Rightarrow 120 = 420 + R_{21} - (30 \times 2)$
$\Rightarrow 120 = 360 + R_{21}$
$\Rightarrow R_{21} = -240$ N
$= 240$ N to the left

(d) $R_{12} = -R_{21}$
$= 240$ N to the right

(e) The net force is still P – friction.
The mass is still 70 kg.
$$\Rightarrow \boldsymbol{a} = \frac{\boldsymbol{F}_{net}}{m}$$
$$= \frac{420 - (70 \times 2.0)}{70}$$
$= 4.0$ m s^{-2} to the right

It is no easier.

39. (a) $m = 75$ kg, $u = -3.2$ m s^{-1}, $\boldsymbol{v} = 0$
Assigning up as positive:
Impulse $= m\Delta\boldsymbol{v}$
$= 75\ (0 - -3.2)$
$= 240$ N s
$= 240$ N s upwards.

(b) $\boldsymbol{F}_{net}\Delta t = m\Delta\boldsymbol{v}$
$\Rightarrow \boldsymbol{F}_{net} \times 0.10 = 240$
$\Rightarrow \boldsymbol{F}_{net} = 2400$ N upwards.
$F_{net} = N - mg$
where N = force ground applies to feet (normal reaction force)
$\Rightarrow 2400 = N - 750$
$N = 3150$ N
$= 3.2 \times 10^3$ N upwards

(c) $u = 0$, $v = -3.2$ m s^{-1}, $a = -10$ m s^{-2}
$v^2 = u^2 + 2ax$
$\Rightarrow (-3.2)^2 = 2 \times -10 \times x$
$$\Rightarrow x = -\frac{(3.2)^2}{20}$$
$= -0.51$ m
Height (assuming feet are motionless relative to the basketballer's centre of mass) is approximately 0.5 m.

40. $m = 1400$ kg, $u = 60$ km h^{-1} $= 16.7$ m s^{-1}, $\boldsymbol{v} = 0$, $t = 0.080$ s

(a) Impulse $= m\Delta\boldsymbol{v}$
$= 1400 \times (0 - 16.7)$
$= 2.3 \times 10^4$ N s opposite to initial direction of motion of the car (2.34×10^4 N s)

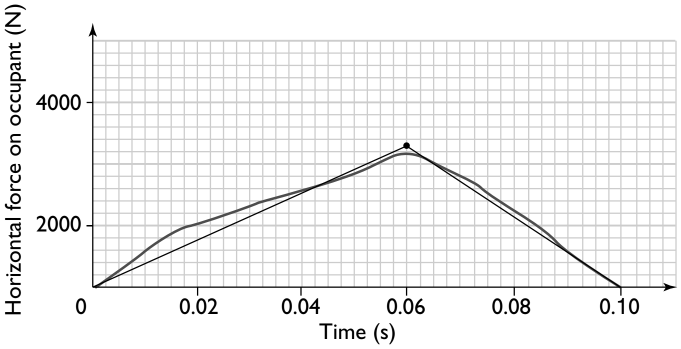
(b) $\boldsymbol{F}_{net}\Delta t = m\Delta\boldsymbol{v}$
$$\Rightarrow \boldsymbol{F}_{net} = \frac{m\Delta\boldsymbol{v}}{\Delta t}$$
$$= \frac{2.34 \times 10^4}{0.080}$$
$= 2.9 \times 10^5$ N opposite to initial direction of motion of the car (2.93×10^5 N).

(c) The deceleration of the driver is the same as the deceleration of the car because the driver is wearing a seat belt.
$$\boldsymbol{a} = \frac{\Delta\boldsymbol{v}}{\Delta t}$$
$$= -\frac{16.7}{0.080}$$
$= -2.1 \times 10^2$ m s^{-2}
$\Rightarrow$ Deceleration $= 2.1 \times 10^2$ m s^{-2}

41. The airbags allow the change in momentum (impulse) of the driver's head to take place over a longer time interval than would be the case if it collided directly with the steering wheel. The average net force on (and the magnitude of the acceleration of) the driver's head is therefore less.

42. The change in momentum (impulse) on the legs takes place over a longer interval, reducing the force exerted by the ground on the knee joint and muscles, tendons and ligaments in the leg.

43. (a) The impulse is the area under the graph. The area is approximately equal to the area of a triangle with a base of 0.10 s and a height of 3200 N.



Area = $\frac{1}{2} \times 0.10 \times 3200$

$\Rightarrow$ Impulse = 160 N s (approximately)

(b) Impulse = $m\Delta v$

$\Rightarrow 160 = 60\Delta v$

$\Rightarrow \Delta v = 2.7$ m s^{-1}

but $v = 0$

$\Rightarrow u = 2.7$ m s^{-1} (approximately)

(c) The impulse on the unbelted occupant is greater than that on the belted occupant (the area under the force versus time graph is clearly greater).
The change in velocity $\Delta \boldsymbol{v}$ is the same for both occupants.
Because impulse = $m\Delta \boldsymbol{v}$, the mass of the unbelted occupant must be greater.
(An estimate of the area under the blue curve shows that the mass of the second occupant is approximately 95 kg.)

(d) The graph describing the force on the occupant with the seat belt shows that the force is applied immediately and is applied for a relatively large amount of time compared with the force applied to the occupant without the seat belt. The occupant without the seat belt experiences no immediate force as she or he continues to move forward at the same speed as the car was moving before impact. The force applied to this occupant increases rapidly to a magnitude greater than the force applied to the occupant with the seat belt. The multiple peaks in force on the second occupant can be explained by multiple impacts with the dashboard or other parts of the car.

44. (a) The impulse is the area under the graph. This can be found most easily by counting the small squares.
The area of each small square = 0.005 s × 100 N = 0.5 N s
The number of small squares under the curve is between 190 and 200.
Impulse = no. of squares × 0.5 N s
= 100 N s upwards (approx.)

(b) Impulse applied by net force = $m\Delta \boldsymbol{v}$

$\Rightarrow$ Impulse applied by floor + impulse due to weight $= m\Delta \boldsymbol{v}$

$\Rightarrow 100 \text{ N s} - mg \times 0.10 = m\Delta \boldsymbol{v}$

$\Rightarrow 100 - 600 \times 0.10 = 60\Delta \boldsymbol{v}$

$$\Rightarrow \Delta \boldsymbol{v} = \frac{100 - 60}{60}$$

$= 0.67$ m s^{-1}
(taking up as positive)

$u = 0 \Rightarrow \boldsymbol{v} = 0.67$ m s^{-1}

(c) $\boldsymbol{F}\Delta t$ = Impulse applied by floor where $\boldsymbol{F}$ = average force applied by floor

$$\Rightarrow \boldsymbol{F} = \frac{100 \text{ N s}}{0.10 \text{ s}}$$

= 1000 N upwards

(d) The normal reaction force is present as the basketballer is initially pushing down on the floor with a force equal to the basketballer's weight.

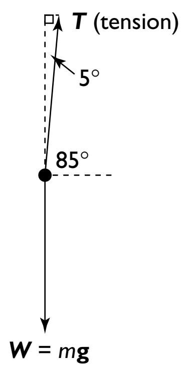
45. Bouncing off during collision results in a greater change in momentum of the cars in a similar or smaller time interval. The rate of change in momentum of the cars, and the resulting net force on the passengers, would therefore be greater ($\boldsymbol{F}\Delta t = m\Delta \boldsymbol{v}$). In low-speed collisions with small vehicles (like dodgem cars) this is not a problem. However, in real cars at typical road speeds more injuries would occur.

46. The driving force, the forward force applied to the tyres by the road, is a reaction to the force applied backwards to the road by the tyres. The size of the driving force is therefore controlled by the driver's use of the accelerator. The driving force acts on all four wheels of a four-wheel-drive vehicle, whereas it acts only on the rear wheels of a rear-wheel-drive car. The front wheels of a rear-wheel-drive car are pushed forward as a result of the driving force on the rear wheels. So the force applied to the front wheels cannot be controlled directly by the driver.

47. All swimmers move forwards in the water because the water pushes them forwards. This is the unbalanced force that provides the acceleration during each stroke (Newton's First Law). The size of the forward force is equal to, and opposite in direction to, the force that the swimmer applies to the water (Newton's Third Law). A freestyle stroke pushes water back with a greater force than a breaststroke stroke. Therefore the forward force is greater for a freestyle swimmer.

48. Student-designed spreadsheet

49.



The forces acting on the yo-yo

The acceleration of the yo-yo in the horizontal direction is the same as the acceleration of the rollerblader.

$\Rightarrow ma$ (horizontal) $= T \sin 5°$ (1)

If the vertical acceleration is zero

$mg = T \cos 5°$ (2)

Dividing (1) by (2)

$$\frac{ma}{mg} = \frac{T \sin 5°}{T \cos 5°}$$

$$\Rightarrow \frac{a}{10} = \tan 5°$$

$\Rightarrow a = 10 \tan 5°$

$= 0.87$ m s^{-2}

50. (a) The Earth and the basketball can be considered to be an isolated system. The total change in momentum of the system is zero. The momentum of the basketball changes. Therefore, the momentum of the Earth must change by the same amount but in the opposite direction. The velocity of the Earth must change as a result. However, the magnitude of the change is very small.

If the mass of the basketball is 0.5 kg and its change in velocity on striking the ground is 5 m s^{-1}:

$\Delta \boldsymbol{p}_b = 0.5 \times 5$

$= 2.5$ kg m s^{-1}

The magnitude of the change in momentum of the Earth is given by:

$\Delta p_E = m_E \Delta v_E$

$\Rightarrow m_E \Delta v_E = 2.5$ kg m s^{-1}

$m_E = 6 \times 10^{24}$ kg

$$\Rightarrow \Delta v_E = \frac{2.5}{6 \times 10^{24}}$$

$= 4 \times 10^{-25}$ m s^{-1}

(b) The concrete wall is firmly attached to the Earth, so the car is effectively colliding with the Earth. The Earth (including the wall) and the car can be considered to be an isolated system. The total change in momentum of the system is zero. The momentum of the car changes. Therefore the momentum of the Earth and concrete wall must change by the same amount but in the opposite direction. The velocity of the Earth and concrete wall must change as a result. However, the magnitude of the change is very small. The distance moved by the concrete wall is also immeasurably small because the force applied to it by the car is very small compared to the forces applied by the ground.

If the mass of the car is 1000 kg and its change in velocity on striking the wall is 10 m s^{-1}:

$\Delta \boldsymbol{p}_C = 1000 \times 10$

$= 10\,000$ kg m s^{-1}

The magnitude of the change in momentum of the Earth and the wall is given by:

$\Delta \boldsymbol{p}_{ew} = m_{ew} \Delta \boldsymbol{v}_{ew}$

$\Rightarrow m_{ew} \Delta \boldsymbol{v}_{ew} = 10\,000$ kg m s^{-1}

$m_{ew} = 6 \times 10^{24}$ kg

$$\boldsymbol{v}_{ew} = \frac{10\,000}{6 \times 10^{24}}$$

$= 2 \times 10^{-21}$ m s^{-1}

Chapter 11

Mechanical interactions

1. By releasing a high-pressure propellant, the astronaut gains momentum in one direction while the propellant gains the same amount of momentum in the opposite direction. The total change in momentum of the astronaut and the contents of his or her backpack is zero.

2. Work = Fx

Unit = N m

but 1 N = 1 kg m s^{-2}

$\Rightarrow$ Unit = kg m s^{-2} × m

= kg m^2 s^{-2}

Kinetic energy = $\frac{1}{2}mv^2$

Unit = kg (m s^{-1})^2

= kg m^2 s^{-2}

3. (a) $m = 750$ kg (estimate)

$v = 60$ km h^{-1} = 16.7 m s^{-1}

$E_k = \frac{1}{2}mv^2$

$= \frac{1}{2} \times 750 \times (16.7)^2$

$= 1 \times 10^5$ J (approx.)

(b) $m = 0.1$ kg (estimate)

$v = 100$ km h^{-1} = 28 m s^{-1} (estimate)

$E_k = \frac{1}{2}mv^2$

$= \frac{1}{2} \times 0.1 \times (28)^2$

$= 4 \times 10^1$ J (approx.)

(c) $m = 65$ kg (estimate)

$v = 10$ m s^{-1} (estimate)

$E_k = \frac{1}{2}mv^2$

$= \frac{1}{2} \times 65 \times (10)^2$

$= 3 \times 10^3$ J (approx.)

(d) $m = 0.01$ kg (estimate)

$v = 1$ mm s^{-1} = 1×10^{-3} m s^{-1} (estimate)

$E_k = \frac{1}{2} \times 0.01 \times (1 \times 10^{-3})^2$

$= 5 \times 10^{-9}$ J (approx.)

4. $W = \Delta E_k$

$= \frac{1}{2}mv^2 + \frac{1}{2}mu^2$

$$= \frac{1}{2} \times 0.058 \times \left(\frac{200}{3.6}\right)^2 - 0$$

$= 90$ J

5. $W = Fx$

$= mgx$

$= 4.0 \times 10 \times 1.5$

$= 60$ J

6. (a) $m = 70$ kg (estimate)

$\Delta h = 2.5$ m (estimate)

$\Delta E_{gp} = mg\Delta h$

$= 70 \times 10 \times 2.5$

$= 2 \times 10^3$ J (approx.)

(b) $m = 30$ kg (estimate)

$\Delta h = 2$ m (estimate)

$\Delta E_{gp} = mg\Delta h$

$= 30 \times 10 \times 2$

$= 600$ J (approx.)

(c) $m = 70$ kg (estimate)

$\Delta h = 0.75$ m (estimate)

$\Delta E_{gp} = mg\Delta h$

$= 70 \times 10 \times 0.75$

$= 500$ J (approx.)

7. None, since there is no displacement in the direction of the force.

8. (a) $W = Fx$
$= 5.0\text{ N} \times 0.10\text{ m}$
$= 0.50\text{ J}$

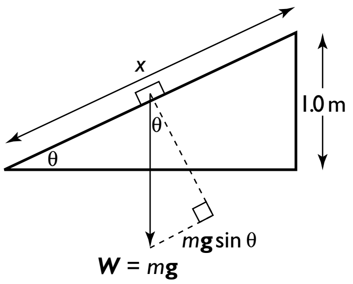
(b) $W = Fx\cos\theta$
$x\cos\theta = 20\text{ cm}$
$= 0.20\text{ m}$
(No calculation of θ needed here since the component of displacement in the direction of the force is shown as 20 cm.)
$\Rightarrow W = 5.0 \times 0.20$
$= 1.0\text{ J}$

(c) Zero, since there is no displacement in the direction of the force.

9. $m = 20\text{ kg}$
$\Delta h = 1.0\text{ m}$
(a) $\Delta E_{gp} = mg\Delta h$
$= 20 \times 10 \times 1.0$
$= 200\text{ J}$

(b) $W = Fx$
$= mgx$
$20 \times 10 \times 1.0$
$= 200\text{ J}$

(c)



To push the crate up the ramp with a constant speed, the applied force must be $mg\sin\theta$.
$W = Fx$

but $\sin\theta = \dfrac{1.0}{x}$

$\Rightarrow x = \dfrac{1.0}{\sin\theta}$

$\Rightarrow W = mg\sin\theta \times \dfrac{1.0}{\sin\theta}$
$= 20 \times 10 \times 1.0$
$= 200\text{ J}$

(d) It is better to use the ramp. Although the amount of work needed to move the crate is the same, the force that needs to be applied to the crate is less if the ramp is used.

10. $P = \dfrac{W}{\Delta t}$

$= \dfrac{\Delta E_k}{\Delta t}$

$= \dfrac{\frac{1}{2}mv^2 - 0}{\Delta t}$

$= \dfrac{\frac{1}{2} \times 0.058 \times \left(\frac{200}{3.6}\right)^2}{4.0 \times 10^{-3}}$

$= 2.2 \times 10^4\text{ W}$

11. $P = \dfrac{W}{\Delta t}$

$= \dfrac{mg\Delta h}{\Delta t}$

$= \dfrac{4.0 \times 10 \times 1.5}{1.2}$

$= 50\text{ W}$

12. (a) $\Delta E_k = \Delta E_{gp}$ (magnitude only)
$\Rightarrow E_k = mg\Delta h$ (since initial $E_k = 0$)
$= 0.160 \times 10 \times 2.0$
$= 3.2\text{ J}$

(b) 32% of 3.2 J = $0.32 \times 3.2\text{ J}$
$= 1.0\text{ J}$ (1.024 J)

(c) Assuming 100% recovery of stored energy
$\Delta E_{gp} = mg\Delta h$
$\Rightarrow 1.024 = 0.160 \times 10 \times \Delta h$

$\Rightarrow \Delta h = \dfrac{1.024}{0.160 \times 10}$
$= 0.64\text{ m}$

13. (a) Provided that friction is negligible, there is no horizontal net force on the system of the mass and the trolley.
⇒ total momentum before = total momentum after
$\Rightarrow 2.0 \times 0.60 = 4.0 \times v$
$\Rightarrow v = 0.30\text{ m s}^{-1}$

(b) There is no horizontal net force on the system of the sand and the trolley.
$\Rightarrow 4.0 \times 0.60 = 2.0 \times v_{sand} + 2.0\ v_{trolley}$
$v_{sand} = 0.60\text{ m s}^{-1}$
as it falls from the moving trolley
$\Rightarrow 4.0 \times 0.60 = 2.0 \times 0.60 + 2.0\ v_{trolley}$
$\Rightarrow 2.0\ v_{trolley} = 1.2$
$\Rightarrow v_{trolley} = 0.60\text{ m s}^{-1}$

14. (a) Total momentum before collision is zero.
Total momentum after collision = 0
$= \boldsymbol{p}_L + \boldsymbol{p}_C$
$\Rightarrow m_L\boldsymbol{v}_L + m_C\boldsymbol{v}_C = 0$
$\Rightarrow m_L \times 1.5 - 50 \times 1.2 = 0$

$\Rightarrow m_L = \dfrac{50 \times 1.2}{1.5}$
$= 40\text{ kg}$

(b) Impulse on Catherine = $m_C\Delta\boldsymbol{v}_C$
$= 50 \times 1.2$
$= 60\text{ N s}$

(c) Impulse on Lauren = –Impulse on Catherine
$= -60\text{ N s}$
Magnitude of impulse = 60 N s

(d) Zero, because there is no external net force acting on the system of the two girls.

(e) They would have greater speeds but still in the same ratio as before. The total momentum would remain as zero as there are no external horizontal forces acting on the girls.

15. (a) Total momentum before collision is given by
$m_N\boldsymbol{v}_N + m_L\boldsymbol{v}_L = 60 \times 2 + 70 \times 0$
$= 120\text{ kg m s}^{-1}$
Total momentum after collision is given by
$m_N\boldsymbol{v}_N + m_L\boldsymbol{v}_L = 60 \times 0 + 70 \times \boldsymbol{v}_L$
$= 70\boldsymbol{v}_L$

Total momentum is conserved

$\Rightarrow 70\boldsymbol{v}_L = 120$

$\Rightarrow \boldsymbol{v}_L = \dfrac{120}{70}$

$= 1.7 \text{ m s}^{-1}$

(b) Impulse on Nick $= m_N \Delta \boldsymbol{v}_N$

$= 60 \times -2$

$= -120$ N s

Magnitude of impulse = 120 N s

(c) Change in momentum = impulse

$= 120 \text{ kg m s}^{-1}$

(d) $\Delta \boldsymbol{p}_L = -\Delta \boldsymbol{p}_N$

since total change in momentum is zero

⇒ magnitude of Luke's change in momentum

$= 120 \text{ kg m s}^{-1}$

(e) They would have different speeds but the total momentum would still be conserved as there are no external horizontal forces acting on the boys.

(f) $\boldsymbol{p}_i = 120 \text{ kg m s}^{-1}$

$\boldsymbol{p}_f = (m_N + m_L)\,\boldsymbol{v}$

$= 120$

$\Rightarrow 130\boldsymbol{v} = 120$

$\Rightarrow \boldsymbol{v} = \dfrac{120}{130}$

$= 0.92 \text{ m s}^{-1}$

16. (a) Total momentum before collision is given by:

$\boldsymbol{p}_C + \boldsymbol{p}_P$

where $\boldsymbol{p}_C$ = momentum of car and driver

$\boldsymbol{p}_P$ = momentum of police car and occupants

$= 1250\boldsymbol{v}_C + 1500 \times 0$

Total momentum after collision:

$(m_C + m_P)\,\boldsymbol{v} = 2750 \times 7$

Total momentum is conserved.

$\Rightarrow 1250 v_C = 19\,250$

$\Rightarrow v_C = 15 \text{ m s}^{-1}$ (15.4 m s^{-1})

(b) Impulse on police car $= m_P \Delta \boldsymbol{v}_P$

$= 1500 \times 7$

$= 10\,500$ N s (1.1×10^4 N s)

in the initial direction of motion of the car.

(c) Impulse on driver $= m_d \Delta \boldsymbol{v}_d$

$= 50 \times (\boldsymbol{v} - u)_d$

$= 50 \times (7 - 15.4)$

$= -420$ N s

$= 420$ N s

opposite to the initial direction of motion of the car.

(d) $\boldsymbol{F}_{net}\Delta t$ = impulse on police car

$\Rightarrow \boldsymbol{F}_{net} = \dfrac{10\,500 \text{ N s}}{0.10 \text{ s}}$

$= 105\,000$ N

Average net force $= 1.1 \times 10^5$ N in initial direction of motion of the car.

17. (a) $\Delta E_k = \Delta E_{gp}$ (magnitude only)

$= mg\Delta h$

$= 1.2 \times 10 \times 20$

$= 240$ J

(b) $E_k = 240$ J (since initial E_k is zero)

$\Rightarrow \frac{1}{2} mv^2 = 240$

$\Rightarrow v^2 = \dfrac{240 \times 2}{1.2}$

$\Rightarrow v = 20 \text{ m s}^{-1}$

18. (a) $W = Fx \cos \theta$

$= 8.0 \times 2.5 \times \cos 20°$

$= 19$ J (18.79 J)

(b) Work done by net force $= \Delta E_k$

$= E_k$ (since initial $E_k = 0$)

$\Rightarrow \boldsymbol{F}_{net}\, x = E_k$

$\Rightarrow E_k = (8 \cos 20° - 7.2) \times 2.5$

$= 0.79$ J (0.794 J)

(c) Zero. There is no component of displacement in the direction of the normal reaction force.

19. (a) $m = 1500$ kg, $u = 50 \text{ km h}^{-1} = 13.9 \text{ m s}^{-1}$

$v = 0$, $x = 0.60$ m

Work done by average net force $= \Delta E_k$

$\Rightarrow F_{net}\, x = E_k$ (initial)

$\Rightarrow F_{net} = \dfrac{\frac{1}{2} \times 1500 \times (13.9)^2}{0.60}$

$= 2.4 \times 10^5$ N

(b) F_{net} (av) $= ma_{av}$

$\Rightarrow a_{av} = \dfrac{2.4 \times 10^5}{1500}$

$= 1.6 \times 10^2 \text{ m s}^{-2}$

(c) Work done by average net force $= \Delta E_k$

$\Rightarrow F_{net}\, x = E_k$ (initial)

$\Rightarrow F_{net} = \dfrac{\frac{1}{2} \times 1500 \times (13.9)^2}{0.10}$

$= 1.449 \times 10^6$ N

$a_{av} = \dfrac{F_{net}}{m}$

$= \dfrac{1.449 \times 10^6 \text{ N}}{1500}$

$= 9.7 \times 10^2 \text{ m s}^{-2}$

(d) The kinetic energy of the car is transformed into potential energy of the materials in the crumple zone which undergo a permanent change in shape. This leaves a smaller amount of kinetic energy to be transferred to the passengers.

(e) One could argue that a large car is safer. For a given force applied by an obstacle or another vehicle, the deceleration of a large car is less than that of a small car. Therefore, the deceleration of the occupants inside is less.

For example, consider a car of mass 1500 kg coming to rest from 20 m s^{-1} when a concrete wall applies a force of 48 000 N to the car.

$a = \dfrac{\boldsymbol{F}_{net}}{m}$

$= -\dfrac{48\,000 \text{ N}}{1500 \text{ kg}}$

$= -32 \text{ m s}^{-2}$

The deceleration of an occupant with a correctly fitted seat belt would be 32 m s^{-2}. Consider a car of mass 1200 kg coming to rest from the same speed when the same force is applied by the wall.

$$a = \frac{\boldsymbol{F}_{net}}{m}$$
$$= -\frac{48\,000 \text{ N}}{1200 \text{ kg}}$$
$$= -40 \text{ m s}^{-2}$$

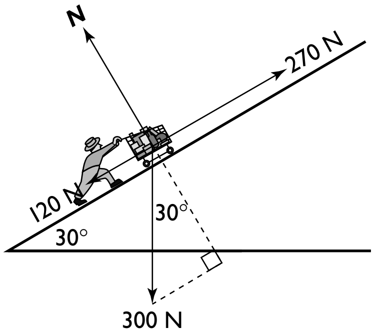
The deceleration of an occupant with a correctly fitted seat belt would be 40 m s^{-2}. Without seat belts, an occupant would strike the interior of a larger car with a smaller relative speed.

Of course, these arguments are not very strong because there are so many other variables related to car design and the nature of the rigid barrier that affect the deceleration of a car.

20. (a) $W = Fx$
$= 270 \times 5$
$= 1.4 \times 10^3$ J (1350 J)

(b) $\Delta E_{gp} = mg\Delta h$
$= 30 \times 10 \times 5 \sin 30°$
$= 750$ J

(c)



Forces acting on the trolley

$W = Fx$
$= mg \times 5.0 \sin 30°$
ΔE_{gp} = work done against the force of gravity
$= 750$ J

(d) $F_{net} = 270 - 300 \sin 30° - 120$
$= 0$
Work done by net force $= F_{net}\, x$
$= 0$

(e) Work done by net force $= \Delta E_k$
$\Rightarrow \Delta E_k = 0$
$\Rightarrow v = 0.50$ m s^{-1}

21. (a) ΔE_k = work done by net force
$\Rightarrow E_k = Fx$ (since net force is constant and initial kinetic energy is zero)
$= 240$ N $\times 8.0$ m
$= 1.9 \times 10^3$ J

(b) $\boldsymbol{F}_{net} = mg \sin 30°$ + friction + air resistance
$\Rightarrow 240 = 50 \times 10 \times \sin 30°$ + friction + air resistance
$\Rightarrow$ friction + air resistance $= 240 - 250$
$= -10$ N
The sum of the friction force and air resistance is 10 N up the slope.

(c) ΔE_k = work done by net force
= area under net force vs distance graph
$= 240 \times 8 + \frac{1}{2} \times (240 + 120) \times 8 + \frac{1}{2} \times 120 \times 4$
$= 1920 + 1440 + 240$
$\Rightarrow E_k = 3600$ J
(since initial kinetic energy is zero)

(d) $\Delta E_{gp} = mg\Delta h$
$= 50 \times 10 \times 20 \sin 30°$
$= 5000$ J

(e) Some of the gravitational potential energy is transformed into thermal energy and sound, due to the frictional force and air resistance.

22. (a) $E_k = \frac{1}{2} mv^2$
$= \frac{1}{2} \times 450 \times (12)^2$
$= 3.2 \times 10^4$ J (32 400 J)

(b) At B, $E_k = 32\,400 + \Delta E_{gp}$ (magnitude)
$= 32\,400 + mg\Delta h$
$= 32\,400 + 450 \times 10 \times 20$
$= 1.224 \times 10^5$ J
$\Rightarrow \frac{1}{2} mv^2 = 1.224 \times 10^5$
$\Rightarrow v = \sqrt{\frac{2 \times 1.224 \times 10^5}{450}}$
$= 23$ m s^{-1}

At C, $E_k = 32\,400 + \Delta E_{gp}$
$= 32\,400 + mg\Delta h$
$= 32\,400 + 450 \times 10 \times 12$
$= 8.64 \times 10^4$ J
$\Rightarrow \frac{1}{2} mv^2 = 8.64 \times 10^4$
$\Rightarrow v = \sqrt{\frac{2 \times 8.64 \times 10^4}{450}}$
$= 20$ m s^{-1} (19.6 m s^{-1})

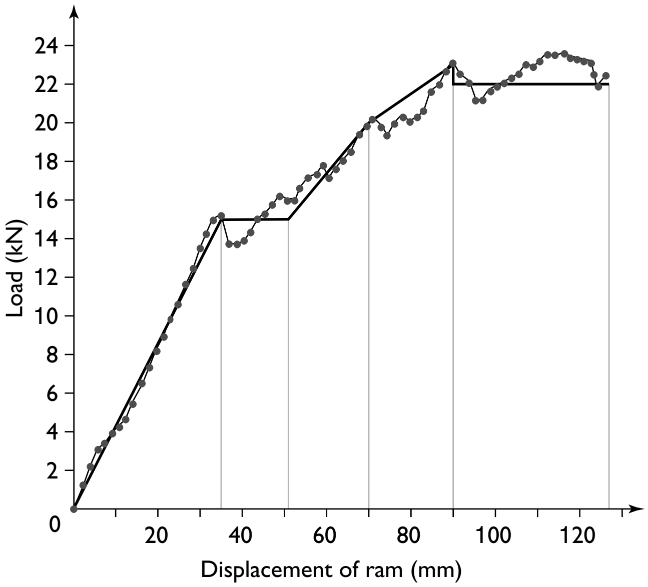
(c) At D, $E_k = 1.224 \times 10^5$ J
Maximum height is achieved when $E_k = 0$
$\Rightarrow \Delta E_k = 1.224 \times 10^5$
$\Rightarrow mg\Delta h = 1.224 \times 10^5$
$\Rightarrow \Delta h = \frac{1.224 \times 10^5}{450 \times 10}$
$\Rightarrow h = 27$ m (since initial height is zero)

23. (a) W = area under F versus x graph
$= 8.9 \times 10^5$ J

(b) W = average force opposing motion $\times\, x$
$= 360 \times 1000$
$= 3.6 \times 10^5$ J

(c) ΔE_k = work done by net force
$= 8.9 \times 10^5$ J $+ 3.6 \times 10^5$ J
$= 5.3 \times 10^5$ J
$\Rightarrow E_k = 5.3 \times 10^5$ J
since initial kinetic energy is zero
$\Rightarrow \frac{1}{2} mv^2 = 5.3 \times 10^5$
$\Rightarrow v = \sqrt{\frac{2 \times 5.3 \times 10^5}{1200}}$
$= 30$ m s^{-1}

24. (a) W = area under load vs displacement graph
This area can be estimated by dividing the area into a number of triangles, trapezia and rectangles. The figure overleaf shows one way in which this can be done.



The unit of area is J since kN × mm = N m
= J

$$W = \frac{1}{2} \times 15 \times 35 + 15 \times (51 + 35)$$

$$+ 19\left(\frac{15 + 20}{2}\right) + 20\left(\frac{20 + 23}{2}\right) + 22 \times 37$$

$= 2080$ J
$= 2.1$ kJ

(b) $mg\Delta h = 2.1 \times 10^3$ J

$$\Delta h = \frac{2.1 \times 10^3}{1400 \times 10}$$

$= 0.15$ m (approx.)

25. So that as little of their kinetic energy as possible is transferred to gravitational potential energy. Subsequently, a greater proportion of their kinetic energy is available to cover the horizontal distance as fast as possible.

26. (a) Work done as the spring expands from maximum compression is equal to the area under the graph.

$\Rightarrow W = \frac{1}{2} \times 0.080 \times 1500$
$= 60$ J

(b) E_k gained = work done by spring
$= 60$ J

(c) E_{gp} gained = E_k lost
$= 60$ J
$\Rightarrow mg\Delta h = 60$

$$\Rightarrow \Delta h = \frac{60}{30 \times 10}$$

$= 0.20$ m

27. As the child falls through the air from maximum height, gravitational potential energy is transformed into kinetic energy.

After the child touches the trampoline after falling through the air, kinetic energy and gravitational potential energy are transformed into elastic potential energy until the trampoline is at maximum extension.

After maximum extension, elastic potential energy is transformed into gravitational potential energy and kinetic energy until contact is lost with the trampoline. After contact is lost, kinetic energy is transformed into gravitational potential energy until maximum height is achieved.

28. (a) $W = Fx$
$= mg\Delta h$
$= 180 \times 10 \times 1.8$
$= 3.2 \times 10^3$ J (3240 J)

(b) $$P = \frac{W}{\Delta t}$$

$$= \frac{3240}{3.0}$$

$= 1.1$ kW (1080 W)

(c) None, since the barbell undergoes no displacement in the direction in which Stefan is applying the force.

29. Force applied to car due to the engine
$= 570$ N + 150 N
$= 720$ N
$P = Fv$ at constant speed
$= 720 \times 20$
$= 14\,400$ J s^{-1}
$= 14$ kW

30. $v = 2.0$ m s^{-1}
stride length = 1.0 m
Man takes 2 strides per second.
$\Delta E_{gp} = 60 \times 10 \times 0.030$ m in each stride
$= 18$ J per stride
ΔE_{gp} in 1 second = 36 J

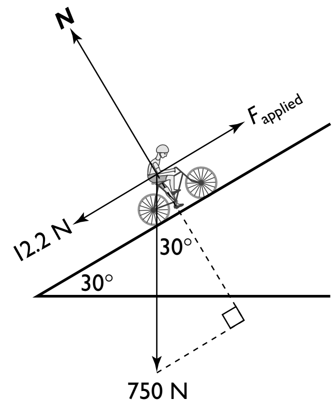
$$P = \frac{\Delta E_{gp}}{\Delta t}$$

$$= \frac{36\text{ J}}{1\text{ s}}$$

$= 36$ W

31. (a) A constant speed is achieved when the force applied to the bicycle to overcome friction and air resistance is 6.5 N + 5.7 N = 12.2 N.
$P = Fv$
$\Rightarrow 56 = 12.2v$
$\Rightarrow v = 4.6$ m s^{-1} (4.59 m s^{-1})

(b) On a slope, the force applied to the bicycle to achieve a constant speed is greater.



The applied force must balance the frictional force and air resistance (a total of 12.2 N) and the component of the weight of the bicycle and its rider down the slope.

$\Rightarrow F_{applied} = 12.2 + mg \sin 30°$
$= 12.2 + 750 \sin 30°$
$= 387.2$ N
$P = Fv$
$= 387.2 \times 4.59$
$= 1777$ W

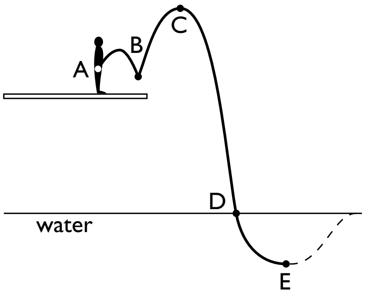
Additional power required = 1777 W − 56 W
$= 1.7 \times 10^3$ W

32. Student-designed spreadsheet

33. (a) Zero. The displacement after one complete revolution is zero. The force applied to keep the toy dog is directed towards the centre of the circle. It is therefore perpendicular to the direction of motion at all times. In other words, there is no displacement in the direction of the applied force. Therefore, no work is done on the dog by the girl.

(b) Although the displacement is not zero after half of a full revolution, there is still no work done on the dog by the girl because the applied force is perpendicular to the direction of motion at all times.

34.



The motion of the centre of mass of a springboard diver

The energy transformations include the following.

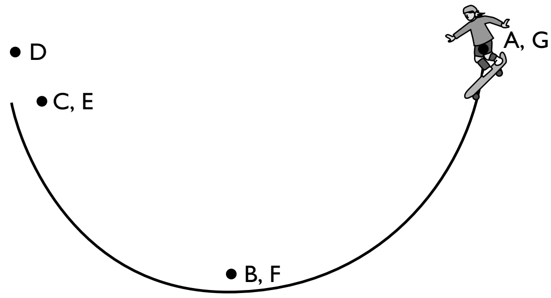
A–B Chemical energy is transformed into elastic potential energy of muscles, tendons and ligaments. Elastic potential energy of muscles is transformed into kinetic energy which is then transformed into gravitational potential energy as the diver rises for the first time. Gravitational potential energy is transformed into kinetic energy as the diver descends to the end of the springboard.

B–C Kinetic energy and some gravitational potential energy is transformed into elastic potential energy of the springboard until the springboard reaches its maximum deflection. The elastic potential energy is transformed into kinetic energy and gravitational potential energy until the diver loses contact with the springboard. Kinetic energy is transformed into gravitational potential energy until the diver reaches maximum height.

C–D Gravitational potential energy of the diver is transformed into kinetic energy until the diver strikes the water.

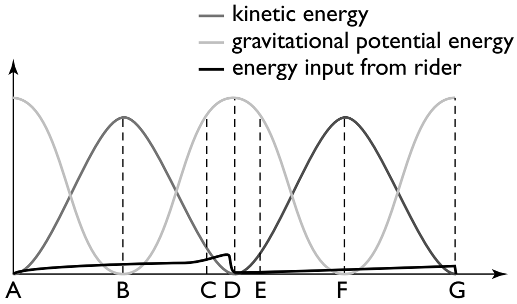
D–E Kinetic energy of the diver is transferred to the water (as kinetic energy) and eventually transformed into thermal energy of the water particles. Some of the diver's kinetic energy is transformed into sound energy.

35. The figure below shows the motion of the centre of mass of the skateboard.



The motion of a skateboard rider completing an 'ollie'

(a) The energy transformations can be displayed with a graph of energy vs time or energy vs position. A sample graph is shown below.



Graph showing energy transformations during an 'ollie'

As the rider moves down and up the slope, gravitational potential energy is transformed to kinetic energy and back again to gravitational kinetic energy. However, the total mechanical energy is not quite conserved and the rider needs to provide some additional energy 'input' to reach the top of the slope and point C. Further energy input is needed from the rider in order to gain the gravitational potential energy required at point D. Gravitational potential energy is then transformed into kinetic energy as the rider returns to point F and transformed into gravitational potential energy at point G. At points A, D and G the rider's kinetic energy is zero.

(b) Between points C and D, the skateboard and the rider are in free fall. They are both travelling at the same speed at point C. Even though they have different amounts of kinetic energy due to their different masses, and therefore gain different amounts of gravitational potential energy, they reach the same height. That is,

$$\frac{1}{2}mv^2 = mg\Delta h$$

$$\Rightarrow v^2 = 2g\Delta h$$

$$\Rightarrow \Delta h = \frac{v^2}{2g}$$

The horizontal components of the speed of both the rider and the skateboard are also the same, as long as air resistance is negligible. The rider therefore needs to make little effort to remain in contact with the skateboard. There is some skill involved in ensuring that the frictional forces made possible by the contact are used to turn the skateboard so that it lands on the ramp before the feet or any other part of the rider's body.